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# Applications of graph theory to an English rhyming corpus

Morgan Sonderegger *

*University of Chicago, Department of Computer Science, 1100 East 58th Street, Chicago, IL 60637, USA*

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**Abstract**

How much can we infer about the pronunciation of a language – past or present – by observing which words its speakers rhyme? This paper explores the connection between pronunciation and network structure in sets of rhymes. We consider the *rhyme graphs* corresponding to rhyming corpora, where nodes are words and edges are observed rhymes. We describe the graph **G** corresponding to a corpus of ∼ 12000 rhymes from English poetry written c. 1900, and find a close correspondence between graph structure and pronunciation: most connected components show community structure that reflects the distinction between full and half rhymes. We build classifiers for predicting which components correspond to full rhymes, using a set of spectral and non-spectral features. Feature selection gives a small number (1–5) of spectral features, with accuracy and *F*-measure of ∼90%, reflecting that positive components are essentially those without any good partition. We partition components of **G** via maximum modularity, giving a new graph, **G**′, in which the "quality" of components, by several measures, is much higher than in **G**. We discuss how rhyme graphs could be used for historical pronunciation reconstruction.


*Keywords:* Rhymes; Graph theory; Complex networks; Poetry; Phonology; English

## 1. Introduction

How can we reconstruct what English sounded like for Pope, Shakespeare, or Chaucer? Pronunciation reconstruction traditionally involves triangulation from several sources; one crucial type of data is rhyming verse (Wyld, 1923). Because rhymes are usually between words with the same endings (phonetically), we might infer that two words which rhyme in a text had identically pronounced endings for the text's author. Unfortunately, this reasoning breaks down because of the presence of "half" rhymes. Consider the following rhymes, from poetry written by William Shakespeare around 1600.[1]

(a) But kept cold distance, and did thence *remove*,
To spend her living in eternal *love*.

(b) And deny himself for *Jove*,
Turning mortal for thy *love*.

* Tel.: +1 773 702 9110; fax: +1 773 702 8487.
*E-mail address:* morgan@cs.uchicago.edu

[1] "A Lover's Complaint" (a,e), *Love's Labour Lost* IV.3 (b), "The Rape of Lucrece"(c), "Venus and Adonis" (d,f).

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(c) But happy monarchs still are fear'd for *love*:
With foul offenders thou perforce must bear,
When they in thee the like offences *prove*:
If but for fear of this, thy will *remove*;

(d) And pay them at thy leisure, one by *one*.
What is ten hundred touches unto thee?
Are they not quickly told and quickly *gone*?

(e) Which fortified her visage from the *sun*,
Whereon the thought might think sometime it saw
The carcass of beauty spent and *done*:
Time had not scythed all that youth *begun*,

(f) What bare excuses makest thou to be *gone*!
I'll sigh celestial breath, whose gentle wind
Shall cool the heat of this descending *sun*:

We write $x$:$y$ when a rhyme is observed between $x$ and $y$, and $x \sim y$ if $x$ and $y$ have the same ending (in a sense made more precise below). One intuitively knows, from experience with songs or poetry, that if $x \sim y$ then it is possible to rhyme $x$ and $y$, and that usually, $x$:$y$ implies $x \sim y$. If we assume that $x$:$y \Rightarrow x \sim y$ always, then from Examples (a)–(c):

love∼Jove∼remove∼prove

and from Examples (d)–(f):

one∼gone∼sun∼done∼begun

However, it turns out that not all words in the first group of rhymes were pronounced the same for Shakespeare, while all words in the second group were.[2] Because of the uncertainty in the implication $x$:$y \Rightarrow x \sim y$, in pronunciation reconstruction rhyming data is only used together with other sources, such as grammar manuals and naive spellings (Wyld, 1923). But these sources are expensive and limited, while rhyming data is cheap and plentiful. If we could somehow make the implication stronger, rhyming data could stand on its own, making reconstruction significantly easier.

This paper attempts to strengthen the implication in two ways: first, by building classifiers to separate half (e.g. (a)–(c)) from full (e.g. (d)–(f)) groups of rhymes, based on the groups' *rhyme graphs*; second, by breaking groups of rhymes into smaller and more full groups, based on the structure of their rhyme graphs. Although the long-term goal of this project is to infer historical pronunciation, this paper uses recent poetry, where the pronunciation is known, to develop and evaluate methods. We first (Sections 2 and 3) introduce rhyme graphs, outline the corpus of poetry used here, and describe its rhyme graph, **G**. In Section 4, we build classifiers for components of **G**, using a set of features which reflect components' graph structure. We then (Section 5) partition components into smaller pieces, giving a new graph $\mathbf{G}'$, and evaluate the quality of rhymes in $\mathbf{G}'$ versus **G**.

## 2. Data

### 2.1. Rhyming corpora

Rhyming corpora have traditionally been used in two ways by linguists interested in phonology. In diachronic phonology, collections of rhymes are traditionally a key tool for pronunciation reconstruction (e.g. Kökeritz, 1953; Dobson, 1968; Wyld, 1936 for English); in this case the focus is on full rhymes, which indicate identity between (parts of) words. In synchronic phonology, rhyming corpora have been used for Japanese song lyrics (Kawahara, 2007), Romanian poetry (Steriade, 2003), English song lyrics (Zwicky, 1976; Katz, 2008), and English poetry (Holtman, 1996; Hanson, 2003; Minkova, 2003).[3] In these cases, the focus is on half rhymes (see below, Section 2.2), which reflect speakers' intuitions about phonological similarity.

[2] General sources on pronunciation around 1600 are (in order of accessibility) (Lass, 1992; Kökeritz, 1953; Dobson, 1968); contemporary phonetic transcriptions (e.g. Danielsson, 1955–1963; Danielsson and Gabrielson, 1972; Kauter, 1930) provide direct evidence.

[3] However, none of the English poetry corpora are electronically available.

Table 1
Summary of authors of rhymes used in the corpus. "Georgian Poets" are contributors to the *Georgian Poetry* anthologies (Marsh, 1916–1922).

| Poet | # Rhymes ($10^3$) | Sources |
|---|---|---|
| A.E. Housman (1859–1936) | 1.52 | Housman (1896, 1922, 1936, 1939) |
| Rudyard Kipling (1865–1936) | 2.60 | Kipling (1889–1896, 1892, 1886) |
| T.W.H. Crosland (1865–1924) | 0.60 | Crosland (1917) |
| Walter de la Mare (1873–1956) | 1.74 | de la Mare (1901–1918) |
| G.K. Chesterton (1874–1936) | 1.29 | Chesterton (1911) |
| Edward Thomas (1878–1917) | 0.52 | Thomas (1917) |
| Rupert Brooke (1887–1915) | 1.05 | Brooke (1915) |
| Georgian Poets (c. 1890) | 3.07 | Georgian Poetry (1911–1919) |

Our use of rhyming corpora differs in several ways. First, we are interested in *both* half and full rhymes. Second, we consider rhyming corpora primarily from the perspective of (applied) graph theory, rather than a linguistic framework. Most importantly, previous work has focused on small subsets of rhymes (usually individual rhymes), or the *local* structure of a corpus; our focus is on *global* structure, as reflected in the corpus' rhyme graph.

Our corpus consists of rhymes from poetry written by English authors around 1900.[4] The contents of the corpus, itemized by author, are summarized in Table 1.

Poetry was obtained from several online databases; most were public-domain sources, one (Twentieth Century English Poetry (Chadwyck-Healey, 2010)) is available through university libraries.

Poems were first hand-annotated by rhyme scheme, then parsed using Perl scripts to extract rhyming pairs. All rhyming pairs implied by a given rhyme scheme were counted, not just adjacent pairs of words. For example, the rhymes counted for (c) above were *love*:*prove*, *prove*:*remove*, and *love*:*remove*, rather than only the first two pairs.

To simplify automatic parsing, we standardized spelling as necessary, for example counting *learned* and *learn'd* as the same word.[5] We also associated each spelling with its most frequent pronunciation; for example, all instances of *wind* were recorded as [wɪnd], corresponding to the most frequent (noun) pronunciation.

### 2.2. Pronunciations, rhyme stems

We took pronunciations for all words in the corpus from cel ex (Baayen et al., 1996), a standard electronic lexicon of British English, as pronounced c. 1988 (in *Everyman's English Dictionary*; Jones et al., 1988). Using 1988 norms for pronunciations around 1900 is an approximation, but a relatively good one, as standard British pronunciation ("RP") has changed relatively little over this period (Wells, 1997). Importantly, rhyming data is only affected by the *relative* pronunciation of words, so that among changes between 1900 and 1988, only mergers and splits (see below, Section 6.1) would affect rhyme quality. The mergers and splits in RP noted by Wells (1997) all affect small sets of words. In examining rhyme graphs for 1900 data using cel ex pronunciations, we only noticed inconsistencies for a few words.

We first define what we mean by "rhyme" and "rhyme stem". Two different definitions of the English rhyme stem (RS) are often used; we call these the *short rhyme stem* and *long rhyme stem*, and consider both in this paper. A word's short rhyme stem is the nucleus and coda of its final syllable, and its long rhyme stem is all segments from the primary stressed nucleus on; Table 2 gives examples. Short and long rhyme stems were found for all words in the corpus, again using cel ex.

Once a definition has been chosen, each word has a unique rhyme stem. A *rhyme* is a pair of two words, $w_1$ and $w_2$, observed in rhyming position in a text.[6] Assuming that a definition of the rhyme stem has been chosen, the rhyme

[4] We use poetry from c. 1900 rather than the present day due to practical considerations. First, rhyming poetry has become less popular over the twentieth century, making it more difficult to find enough rhymes for a sizable corpus. Second, much recent poetry is still under copyright, making it harder to obtain electronic versions of poems.

[5] Spelling variants in poetry can indicate different intended pronunciations, a possibility we are abstracting away from.

[6] We assume that the rhyme scheme, which specifies which words in a stanza rhyme, is known. For our corpus, rhyme schemes were coded by hand.

Table 2
Examples of short and long rhyme stems. 'IPA' indicates a word's pronunciation (from cel ex) in International Phonetic Alphabet transcription.

| Word | IPA | Short RS | Long RS |
|---|---|---|---|
| bat | bæt | æt | æt |
| cement | sɨ.mɛńt | ɛnt | ɛnt |
| England | ɨ́ŋ.glənd | ənd | ɨŋgləənd |

Table 3
Examples of full rhymes and half rhymes, for short and long rhyme stems. For example, the short rhyme stems of *travel* and *gobble* are identical (full rhyme), but their long rhyme stems are not (half rhyme). It is not possible for two words to have identical long rhyme stems (full rhyme) but different short rhyme stems (half rhyme).

| | | Short RS | |
|---|---|---|---|
| | | Full rhyme | Half rhyme |
| Long RS | Full rhyme | *Parting*:*darting* | N/A |
| | Half rhyme | *Travel*:*gobble* | *Portrait*:*parted* |

is *full* if the rhyme stems of $w_1$ and $w_2$ are the same, and *half* otherwise.[7] Table 3 shows examples of full rhymes and half rhymes between pairs of bisyllabic words, for both rhyme stem definitions.

## 3. Rhyme graphs

### 3.1. Notation

We first introduce formalism for associating a rhyming corpus with a weighted graph, the *rhyme graph*. The general idea of using graph theory to study sets of rhymes has to our knowledge been proposed once before, in the 1970s by Joyce (1977, 1979), with a somewhat different formalism and smaller scope than here.[8]

A rhyming corpus consists of a set $\mathcal{R}$ of rhymes, defined as (unordered) pairs of words. We write a rhyme between words $v_i$ and $v_j$ as $\{v_i, v_j\}$. Let $V$ be the set of all words (word types) which occur in some word pair, and let $n = |V|$. Let $n_{ij}$ be the number of times the rhyme $\{v_i, v_j\}$ is observed, and assume $n_{ii} = 0$ (there are no self-rhymes). Let $d_i$ be the *degree* of $v_i$:

$$d_i = \sum_j n_{ij}$$

Let $a_i$ be the number of edges connected to $v_i$: $a_i = |\{v_j | n_{ij} > 0\}|$.[9]

We associate with a rhyme corpus two types of weighted graph $G = (V, E, W)$, the *rhyme graph*, where $E = \{\{v_i, v_j\} | v_i, v_j \in V, n_{ij} > 0\}$, and $w_{ij}$ is the weight of the edge between words $w_i$ and $w_j$:

1. Unnormalized weights: $w_{ij} = n_{ij}$
2. Normalized weights: $w_{ij} = n_{ij}/\sqrt{d_i d_j}$.

Let $d_i'$ be the *weighted degree* of node $i$:

$$d_i' = \sum_j w_{ij}$$

[7] These are also known as "perfect" and "imperfect" rhymes.
[8] Joyce considers the late Middle English poem *Pearl* (2222 rhymes), uses directed rather than undirected graphs, and shows several components of the rhyme graph for *Pearl*.
[9] In the unweighted case, where $n_{ij}$ is 0 or 1, $a_i$ would be the degree of $v_i$.

We use "vertex" and "word" interchangeably, and use "edges" to mean "pairs of vertices $\{w_i, w_j\}$ such that $w_{ij} \neq 0$". By "component of $G$" we mean "connected component of $G$": a subgraph in which any node can be reached from any other node (via some path consisting of positive-weight edges), and to which no additional nodes or edges from $G$ can be added while preserving this property.

### 3.2. The rhyme graph **G**

Parsing the corpus gave 12387 rhymes (6350 distinct types), consisting of pairs of 4464 words. About half these words (2024) only occur in one rhyme. Types which only appear once in a corpus are often called *hapax legomena* (or *hapaxes*). We sanitized the data using two steps, which empirically seem to make the structure of the rhyme graphs more transparent.

1. All rhymes including hapaxes were excluded from the corpus, for a total of 10363 rhymes (4326 types) between 2440 words.
2. We removed all components of fewer than 6 words (after removing hapaxes) leaving 9388 rhymes (4059 types) between 1785 words.

A component size cutoff is motivated by practical concerns. The spectral features used below (Section 4) are measures of the quality of the best partition of a component into two non-trivial ($\geq$2 vertices), connected subgraphs. For components of size <4 there are no non-trivial partitions, so some size cutoff is needed. The choice of 6 is arbitrary. Below (Section 4.3), we consider the effect of varying the component size cutoff on performance.

We denote the rhyme graph corresponding to the corpus as **G**. **G** has 70 connected components; we call this set $\mathcal{C}$. To visualize the graphs corresponding to rhyme data, we use GraphViz, an open-source graph visualization package (AT&T Research, 2006; Gansner and North, 1999). In all graphs shown here, colors indicate pronunciations of final syllable vowels, a necessary but not sufficient condition for full rhyme (for both long and short rhyme stems); and the $\{w_i, w_j\}$ edge is labeled with $n_{ij}$. Because the whole graph is too large to show, we illustrate what **G** looks like through some example components.

*Common component types:* Many components consist entirely (Fig. 1(c)) or mostly (Fig. 1(d)) of words with a single rhyme stem. When components contain more than one rhyme stem, they often consist of two or more dense clusters largely corresponding to different rhyme stems, with relatively few edges between clusters. For example, the components in Fig. 1(a) and (b) consist of two well-defined clusters, and Fig 5(c) shows a component with 10 clusters, discussed further below. These two types of components make up much of the graph, though some are not as clean as the examples shown. In components made of several clusters, the clusters seem to correspond to words primarily sharing a single rhyme stem (and connected by full rhymes), with edges between these clusters corresponding to half rhymes. This intuition is confirmed below (Sections 4 and 5) in the classification and partitioning tasks.

*Less common component types:* Two other types of components occur, though less frequently. Components such as Fig. 2(a) contain many edges corresponding to half rhymes between words with similar spellings, or *spelling rhymes* (Wyld, 1923). Components such as Fig. 2(b) contain many edges which correspond to full rhymes if a special literary pronunciation is used for one word (usually ending in a suffix).[10] For example, reading *-ness* as [nɛs] would make all half rhymes in Fig. 2(b) full, and reading *-ity* as [ɪtaɪ] would make rhymes such as *sanity*:*fly* full (using short rhyme stems). We call such cases *poetic pronunciation conventions* (PPCs). PPCs often (as in Fig. 2(b)) result in spelling rhymes, but not always (as in *sanity*:*hi*).

### 3.3. Summary

We have introduced rhyme graphs, the corpus, and its rhyme graph **G**. We found that most of its components consist of one or more well-defined clusters, reflecting the real pronunciation of words, while the rest reflect poetic usage. We now make practical use of the relationship between structure and pronunciation seen in most components of **G**.

[10] In most cases, these pronunciations are "literary" in the sense that they have not been used in colloquial English for centuries (Wyld, 1936).

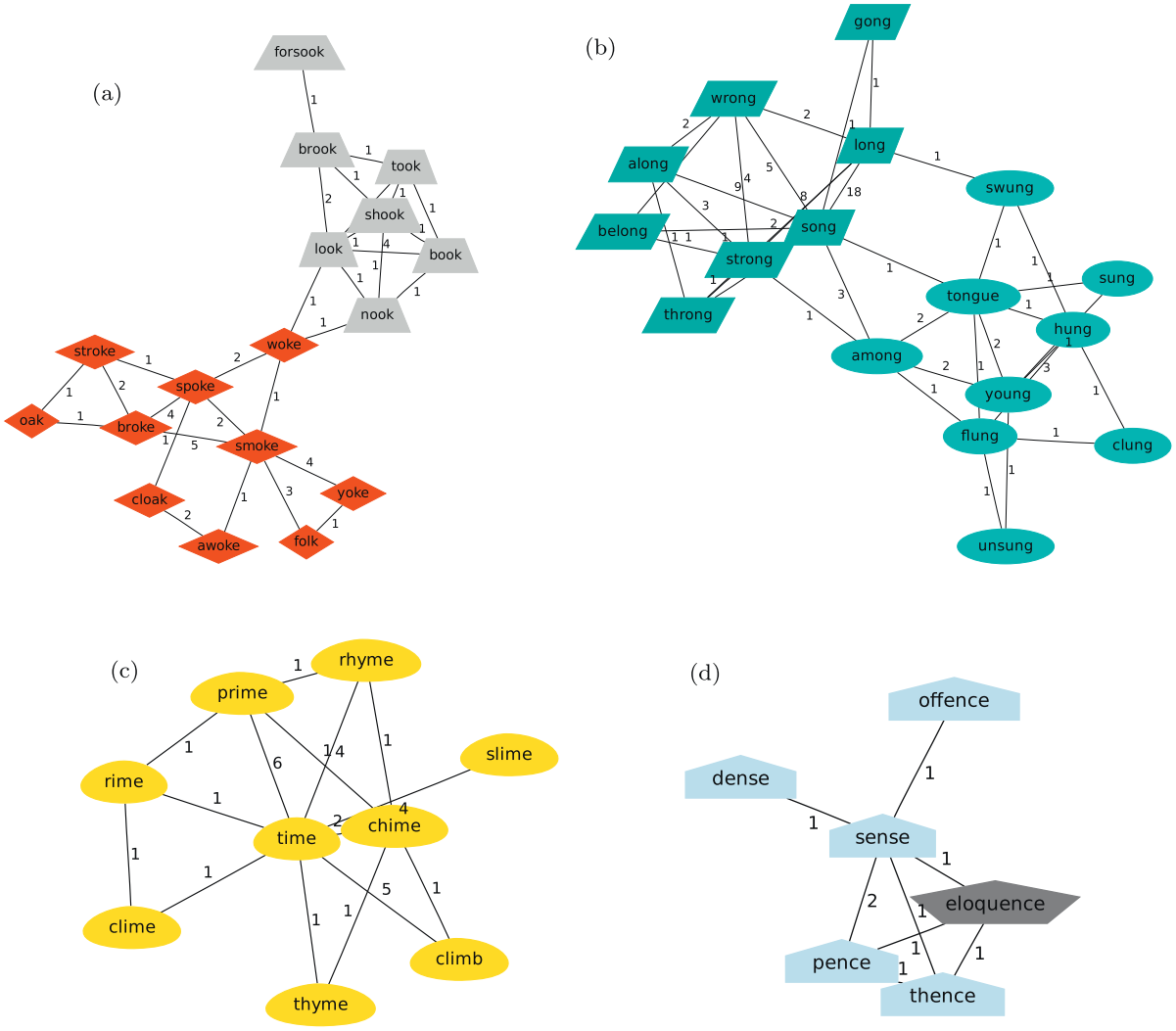



Fig. 1. Examples of common component types from **G**. (a and b) Full-rhyme clusters connected by half rhyme edges. (c) Single full rhyme cluster. (d) Full-rhyme cluster including a small percentage of outliers. In these and subsequent rhyme graph components, colors correspond to the final vowel, as listed in cel ex.

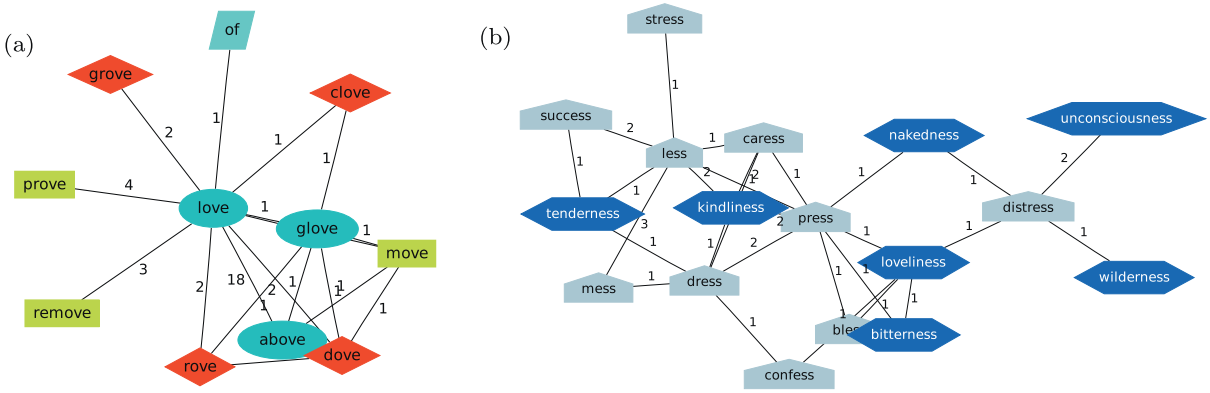



Fig. 2. Examples of less common component types from **G**: (a) spelling rhymes and (b) poetic pronunciation convention.

## 4. Classification

Some components of **G** contain mostly words corresponding to a single rhyme stem; other components do not. In this section, we attempt to predict which group a given component falls into, using features derived from its

graph structure. We first describe the feature set and a binary classification task implementing the intuitive notion of component "goodness", then train and evaluate several classifiers for this task over the components of **G**. We find that the "goodness" of a component – corresponding to whether it contains half-rhymes – can be largely determined from graph structure alone, independent of any information about pronunciation or spelling.

### 4.1. Feature set

Let $G = (V, E, W)$ be a connected component of **G**, where $W$ are the unnormalized weights ($w_{ij} = n_{ij}$), with $n$ nodes ($v_1, \ldots, v_n$), and $m$ edges, corresponding to $\{v_i, v_j\}$ pairs with $w_{ij} > 0$. Define $n_{ij}$, $d_i$, $w_{ij}$, $d'_i$, and $a_i$ as above (Section 3.1). In addition, we need a matrix of distances, $d$, between nodes in a component. Because higher weight should correspond to smaller distance, we use

$$d_{ij} = \begin{cases} 1/w_{ij} & \text{if } w_{ij} \neq 0 \\ \infty & \text{otherwise} \end{cases}$$

We define 17 features describing the structure of $G$, of two types. *Non-spectral features* are properties of graphs (e.g. diameter, maximum clique size) often used in research on social networks (Wasserman and Faust, 1994; Scott, 2000), and complex networks more generally (Newman, 2003; Bornholdt and Schuster, 2003). *Spectral features* are based on the eigenvalues of the Laplacian of $G$, which are intimately related to $G$'s structure (Chung, 1997).[11]

#### 4.1.1. Non-spectral features

We first define some non-spectral properties of graphs often used in network research.

Let $g$ be the matrix of *geodesics* using distances $d$: $g_{ij}$ is the length of the shortest path between vertices $i$ and $j$. The *vertex betweenness centrality* of $v \in V$ is the percentage of geodesics that include $v$.

Considering $G$ as an unweighted graph (with $\{v_i, v_j\} \in E$ if $w_{ij} > 0$), the *clustering coefficient* of $v_i \in V$ is the number of edges between neighbors of $v$, divided by the total number possible:

$$C(v_i) = \frac{|\{v_j, v_k\} \in E : v_i \sim v_j \sim v_k|}{|\{v_j, v_k\} \in E : v_i \sim v_j, v_i \sim v_k|}$$

We define 10 non-spectral features:

- `mean/max nzd degree`: Mean/maximum of the $a_i$, divided by $n - 1$.
- `edge rat`: $(m/(n(n-1)/2))$, the fraction of all possible edges present.
- `max clique size nzd`: Fraction of vertices in the largest clique of $G$.
- `max vtx bcty`: Maximum vertex betweenness centrality.
- `diameter`: Maximum of the $g_{ij}$.
- `mean shortest path`: Mean of the $g_{ij}$.
- `radius`: The minimum eccentricity of a vertex, where eccentricity $(v_i) = \max_{v_j \in V} g_{ij}$.
- `ccoeff`: Mean clustering coefficient for vertices in $V$.
- `log size`: log $(n)$

In addition, `log size` is $z$-scored across components, and `diameter` is divided by its maximum value across components. With these normalizations made, no non-spectral feature transparently depends on the absolute size of a component. This makes the non-spectral features more comparable to spectral features, which are intuitively related to the "shape" of a graph, not its size.

#### 4.1.2. Spectral features

We outline the relationship between the eigenvalues of $G$ and measures of how "cuttable" $G$ is, then define features based on this connection.

[11] A larger feature set including 18 other non-spectral features was initially tried, but did not give better classification performance. The non-spectral features used here are the most predictive features from the larger set.

*Graph Laplacians:* There are several ways to define the Laplacian of a graph, which in general yield "different but complementary information" about its structure (Chung and Lu, 2006). We use three versions here.

Let $A$ be the adjacency matrix of component $G$. Let $N$ and $D'$ be the diagonal matrices with diagonal elements $a_i$ and $d'_i$, respectively. The *unweighted, unnormalized Laplacian* is

$$L_{00} = N - A$$

The *weighted, unnormalized Laplacian* is

$$L_{10} = D' - W$$

The *weighted, normalized Laplacian* is

$$L_{11} = D'^{-1/2} L_{10} D'^{-1/2}$$

However, it is defined, the Laplacian's eigenvalue spectrum is closely related to many properties of the underlying graph ($G$); the study of this connection is *spectral graph theory* (Chung, 1997). Graph eigenvalues are essential because they figure in a variety of bounds on quantities of interest about the underlying graph, such as finding the "best" partition, which are often provably NP-hard to compute. Our spectral features are based on several such bounds.

It can be quickly checked that $L_{00}$, $L_{10}$ and $L_{11}$ (a) are positive semi-definite, which implies their eigenvalues are real and positive (b) have smallest eigenvalue 0. Let $\lambda_{00}$, $\lambda_{10}$, and $\lambda_{11}$ be the second-smallest eigenvalues. Let $\lambda'_{00}$ be the largest eigenvalue of $L_{00}$, and denote $\mu_{00} = 2\frac{\lambda_{00}}{\lambda_{00}+\lambda'_{00}}$.

These eigenvalues can be used to bound several measures of the "cuttability" of a graph. Let $E(S, \bar{S})$ be the set of edges between a subset $S \subset V$ and its complement, and define $vol(S) = \sum_{v \in S} a_i$. The *Cheeger constant* of $G$ is

$$h_G = \min_{S \subset G} \frac{|E(S, \bar{S})|}{\min(\text{vol}(S), \text{vol}(\bar{S}))} \tag{1}$$

Intuitively, the Cheeger constant corresponds to a bipartition of $G$ which balances two conflicting goals: make the two pieces as equal in size as possible, and separated by as few edges as possible.[12] It can be shown (Chung, 1997) that

$$\frac{\lambda_{00}}{2} \leq h_G \leq \sqrt{1 - (1 - \lambda_{00})^2} \tag{2}$$

For the weighted case, define $E(S, \bar{S})$ to be the sum over weights of edges between $S$ and $S'$, and let $\text{vol}(S) = \sum_{i \in S} d'_i$. The Cheeger constant is then defined as in (1). Lower (Mohar, 1997, p. 32) and upper (Friedland and Nabben, 2002, p. 10) bounds on $h_G$ using $\lambda_{10}$, analogous to the unweighted case in (2), are:

$$\frac{\lambda_{10}}{2} \leq h_G \leq \sqrt{1 - \left(1 - \frac{\lambda_{10}}{\delta}\right)^2} \tag{3}$$

where $\delta$ is the minimum weighted degree of a vertex ($\min d'_i$).

Similarly, $h_G$ can be bounded using $\lambda_{11}$:

$$\frac{\lambda_{11}}{2} \leq h_G \leq 2\sqrt{\lambda_{11}} \tag{4}$$

Finally, we consider a different measure of the geometry of $G$. For the unweighted version of $G$ (adjacency matrix $N$), given a subset $X \subset V$, define the *vertex boundary* $\delta(X)$ to be the vertices not in $X$, but adjacent to a vertex in $X$. Then for any subset, the ratio between its "perimeter" and "area" can be bounded from below (Chung, 1997):

$$\frac{\text{vol}(\delta X)}{\text{vol}(X)} \geq \frac{1 - (1 - \mu_{00})^2}{1 + (1 - \mu_{00})^2} \tag{5}$$

Intuitively, if the perimeter/area ratio can be lower-bounded for all $X$, as in a circle, there is no good cut of $G$.

[12] The Cheeger constant is also called "conductance".

Based on these relationships between a graph's eigenvalues and its "cuttability", we define 7 spectral features corresponding to the bounds in (2)–(5):

- `cut lower bound 1`: $\frac{\lambda_{00}}{2}$
- `cut upper bound 1`: $\sqrt{1-(1-\lambda_{00})^2}$
- `cut lower bound 2`: $\frac{\lambda_{10}}{2}$
- `cut upper bound 2`: $\sqrt{1-\left(1-\frac{\lambda_{10}}{\delta}\right)^2}$
- `cut lower bound 3`: $\frac{\lambda_{11}}{2}$
- `cut upper bound 3`: $2\sqrt{\lambda_{11}}$
- `subset perim/area bound`: $\frac{1-(1-\mu_{00})^2}{1+(1-\mu_{00})^2}$

### 4.2. Experiments

We now describe a binary classification task involving this feature set, as well as subsets of these features.

#### 4.2.1. Binary classification task

For both short and long rhyme stem data, we wish to classify components of the rhyme graph as "positive" (consisting primarily of true rhymes) or "negative" (otherwise). As a measure of component goodness, we use the percentage of vertices corresponding to the most common rhyme stem, denoted most frequent rhyme stem percentage (`MFRP`). The pronunciation of rhyme stems was manually determined from cel ex, as discussed above (Section 2.2). Intuitively, if a component is thought of as made up of vertices of different colors corresponding to different rhyme stems, the `MFRP` is the percentage of vertices with the most common color.[13] For example, the component in Fig. 1(b) has `MFRP` =52.9 (corresponding to [ʊŋ] rhyme stems). To turn this into a binary classification task, we choose a threshold value, `threshMFRP`, arbitrarily set to 0.85 in tests described here. We fix `threshMFRP` for purposes of exposition; below (Section 4.3), we consider the effect of varying `threshMFRP` on performance. We wish to separate components with `MFRP<threshMFRP` from components with `MFRP>threshMFRP`.

As a measure of how predictive different features are for classification, Table 4 shows the information gain associated with each feature, for long and short RS data (see, e.g. Russell and Norvig, 2002). In both cases, there is a clear asymmetry between spectral and non-spectral feature predictiveness: every spectral feature is more predictive than all non-spectral features.

#### 4.2.2. Feature sets

Given that we have a large number of features relative to the number of components to be classified, and that many features are strongly correlated, we are in danger of overfitting if the full feature set is used. For each of long and short RS data, we define two smaller feature sets.

The first is the subset given by correlation-based feature selection (CFS; Hall, 1999), a standard feature-selection strategy in which a subset is sought which balances features' predictiveness with their relative redundancy. We used the implementation of CFS in Weka (Witten and Frank, 2005), an open-source machine learning package. Table 4 shows the optimal CFS subsets for long and short RS data.

The second subset used, for each of the short and long RS datasets, is simply the most predictive feature: `cut lower bound 1` for short RS data, and `subset perim/area bound` for long RS data.

[13] A node's color in the rhyme graph components shown here corresponds to its final vowel nucleus, not its rhyme stem. However, within an *individual* component, there is usually at most one rhyme stem with a given final vowel nucleus, so that each color can be thought of as representing one rhyme stem. We use colors to represent final vowels, rather than rhyme stems, because the number of different rhyme stems (∼50) makes it impractical to represent each one by a visually distinct color.

Table 4
Information gain of features for short and long rhyme stem classification. (Dashed line divides spectral and non-spectral features.) Feature subsets given by correlation-based feature selection are marked with asterisks. The maximally predictive features are `cut lower bound 1` for short RS data and `subset perim/area bound` for long RS data.

| Short RS | Info. gain | Long RS | Info. gain |
|---|---|---|---|
| `cut upper bound 1*` | 0.57 | `subset perim/area bound*` | 0.54 |
| `cut lower bound 1*` | 0.57 | `cut lower bound 2*` | 0.53 |
| `subset perim/area bound*` | 0.54 | `cut lower bound 3*` | 0.50 |
| `cut upper bound 3` | 0.49 | `cut upper bound 3` | 0.50 |
| `cut lower bound 3*` | 0.49 | `cut lower bound 1*` | 0.46 |
| `cut lower bound 2` | 0.45 | `cut upper bound 1` | 0.46 |
| `cut upper bound 2` | 0.42 | `cut upper bound 2` | 0.38 |
| `max nzd degree` | 0.29 | `mean nzd degree` | 0.28 |
| `mean nzd degree` | 0.26 | `edge rat` | 0.28 |
| `edge rat` | 0.26 | `max vtx bcty*` | 0.24 |
| `radius*` | 0.23 | `radius` | 0.23 |
| `mean shortest path` | 0.22 | `mean shortest path` | 0.22 |
| `max clique size nzd` | 0.22 | `diameter` | 0.21 |
| `diameter` | 0.22 | `max clique size nzd` | 0.20 |
| `log size` | 0.19 | `max nzd degree` | 0.19 |
| `max vtx bcty*` | 0.18 | `log size` | 0.17 |
| `ccoeff*` | 0.18 | `ccoeff*` | 0.15 |

Table 5
10-Fold cross-validated accuracies (percentage correct) and *F*-measures for several classifiers over components of **G**, for short (top) and long (bottom) rhyme stems. For each rhyme stem type, CFS subset and most predictive feature given in Table 4.

| | | *Accuracy* | | | | | *F-measure* | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | Base | KNN-5 | KNN-10 | CART | SVM | Base | KNN-5 | KNN-10 | CART | SVM |
| Short | All features | 55.7 | **88.6** | 85.7 | 81.4 | 85.7 | 71.6 | **89.5** | 86.8 | 83.5 | 87.5 |
| | CFS subset | 55.7 | 87.4 | 85.7 | 81.4 | **88.6** | 71.6 | 88.6 | 87.2 | 83.5 | **90.2** |
| | Most pred. feature | 55.7 | 87.1 | **90.0** | 85.7 | **90.0** | 71.6 | 88.6 | **91.1** | 85.7 | 88.9 |
| Long | All features | 47.1 | **85.7** | 84.3 | **85.7** | 81.4 | 47.1 | 84.8 | 82.5 | **85.3** | 80.0 |
| | CFS subset | 47.1 | 82.9 | 85.7 | 85.7 | **87.1** | 47.1 | 81.8 | 84.4 | 85.3 | **85.7** |
| | Most pred. feature | 47.1 | 85.7 | 84.3 | **88.6** | 84.3 | 47.1 | 85.3 | 83.1 | **88.6** | 82.0 |

#### 4.2.3. Classifiers

There are 33 positive/37 negative components for long rhyme stems, and 39 positive/31 negative components for short rhyme stems. As a baseline classifier `Base`, we use the classifier which simply labels all components as positive.

We use three non-trivial classifiers: *k*-nearest neighbors, classification and regression trees (Breiman et al., 1984) and support vector machines (Vapnik, 1995). We used Weka's versions of these classifiers, with the following settings:

- `KNN-5` /`KNN-10`: Classify by 5/10 nearest neighbors, using Euclidean distance.
- `CART`: Binary decision tree chosen using minimal cost-complexity pruning, with five-fold internal cross validation (`numFoldsPruning` =5), minimum of 2 observations at terminal nodes.
- `SVM`: Support vector machine trained using sequential minimal optimization (Platt, 1999). All features normalized, $C = 1$ (complexity parameter), linear homogeneous kernel.

#### 4.2.4. Results

Table 5 shows classification results using 10-fold cross-validation, stratified by dataset (short vs. long RS), feature set, and classifier. Classifiers' performances are given as accuracies (percentage of instances correctly labeled) and *F*-measures (harmonic mean of precision and recall).

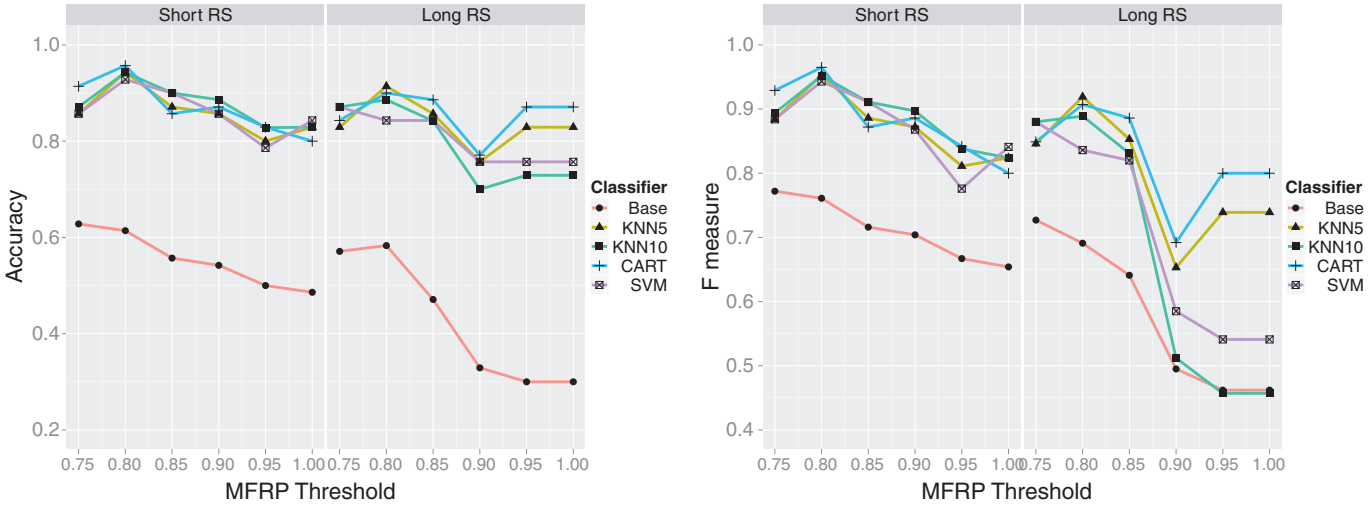



Fig. 3. 10-Fold cross-validated accuracies and *F*-measures for several classifiers over components of **G**, for short and long rhyme stems, as `threshMFRP` is varied; the component size cutoff is fixed at 6. Only the most predictive feature is used for classification.

Unsurprisingly, in all cases the non-trivial classifiers perform better than the baseline classifier. Performance using either feature subset is significantly better than for the full feature set, suggesting overfitting or badly predictive features. Performance (using non-trivial classifiers) is better for short rhyme stems than for long rhyme stems, though the difference is only statistically significant for *F*-measures.[14] This could be taken to argue that the corpus' rhymes are better described using short rhyme stems.

It is striking that across classifiers, performance measures, and rhyme stem types, performance using only *one feature* (the most predictive feature) is not clearly worse than performance using a subset of features given by a feature selection procedure (CFS). For both long and short rhyme stems, the most predictive feature (and the first several most predictive features generally, as seen in Table 4) is spectral. Classifying components then comes down to a single feature which can be interpreted in terms of graph structure: more cuttable components (lower $\lambda$) are classified as negative, while less cuttable components (higher $\lambda$) are classified as positive.

### 4.3. Sensitivity analysis

To construct the dataset used in the classification experiments, two free parameters were fixed: the threshold `MFRP` (`threshMFRP`), and the component size cutoff (CSC). Varying `threshMFRP` changes which components have positive labels, while changing CSC changes the number of components in the dataset. We now briefly check how sensitive the experimental results summarized in Table 5 are to varying these parameters, to make sure that the particular values chosen are not responsible for the good performance of our classifiers. We re-ran all experiments for the most predictive feature condition, changing the dataset by varying one of `threshMFRP` and CSC at a time.[15]

Because the goal of the classification task is to distinguish components which correspond mostly to a single rhyme stem from those which do not, only values of `threshMFRP` relatively near 1 make sense. (It would not make sense, for example, to define "good" components as those with `MFRP`>0.5.) We consider $t$*hreshMFRP* $\in [0.75, 1]$. Fig. 3 shows accuracies and *F*-measures for classification on data resulting from values of `threshMFRP` in this range, with CSC kept fixed at 6. For short rhyme stem experiments, accuracies for non-trivial classifiers range between 79 and 95%, and *F*-measures range between 78 and 97%. For long rhyme stem experiments, accuracies range between 70 and 92%, while *F*-measures range greatly, between 46 and 92%. For both short and long rhyme stems, for all classifiers, two points are important: no classifier achieves its best performance at `threshMFRP` =0.85, and there is a range of values including `threshMFRP` =0.85 within which performance varies little.

[14] $p = 0.21$ for accuracies, $p = 0.01$ for *F*-measures, Wilcoxson paired rank-sum test.
[15] Which feature was most predictive changed as `threshMFRP` and CSC were changed, but was always a spectral feature.

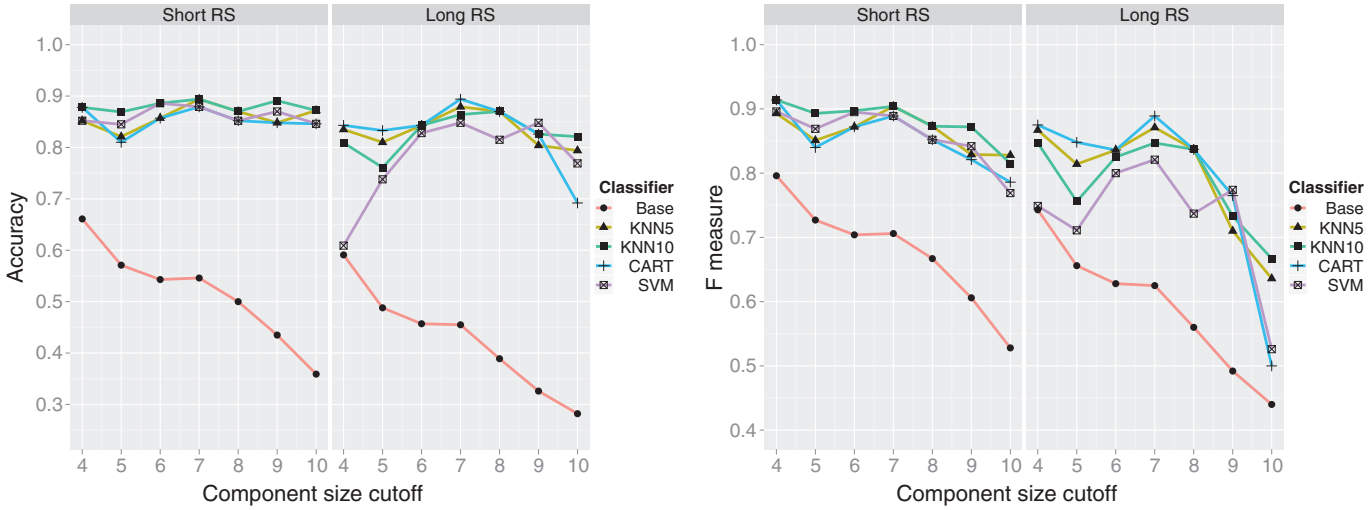



Fig. 4. 10-Fold cross-validated accuracies and *F*-measures for several classifiers over components of **G**, for short and long rhyme stems, as the component size cutoff is varied; `threshMFRP` is fixed at 0.85. Only the most predictive feature is used for classification.

As discussed above (Section 3.2), CSC must be at least 4 for spectral features to make sense for all components; we consider CSC $\in [4, 10]$. Fig. 4 shows accuracies and *F*-measures for classification on data resulting from values of CSC in this range, with `threshMFRP` kept fixed at 0.85. For short rhyme stem experiments, accuracies for non-trivial classifiers range between 81 and 90%, and *F*-measures range between 77 and 92%. For long rhyme stem experiments, accuracies range between 61 and 90%, while *F*-measures range between 51 and 89%. The same two points hold as for the `threshMFRP` case: no classifier achieves its best performance at CSC=6, and there is a range of values including CSC=6 within which performance varies little.

Classification performance can change significantly as `threshMFRP` and CSC are varied, especially for experiments using long rhyme stems. However, performance is not optimal for the fixed values of `threshMFRP` and CSC used above, and for most experiments, the change in performance when these parameters are varied near their fixed values is small. We can thus conclude that the good performance obtained in Section 4.2 is not a result of the particular values used for `threshMFRP` and CSC.

### 4.4. Summary

We have found that spectral features are more predictive of component goodness than non-spectral features; and that although different spectral features in principle provide independent information about components, classifiers using a single spectral feature have 85–90% accuracy, significantly better than a baseline classifier, and in line with classifiers trained on an optimized subset of features. We have also shown that this performance is not an artifact of the particular values chosen for two free parameters used to construct the dataset. For both short and long rhyme stems, the single spectral feature corresponds to how "cuttable" each component is. We have thus confirmed the intuition that by and large, the bad components are those for which a good partition exists. We now see whether such good partitions can be used to increase the quality of the dataset itself.

## 5. Partitioning

For each connected component of **G**, we would like to find the best partition into several pieces. The optimal number of pieces is not known beforehand, and if no good partition exists, we would like to leave *C* unpartitioned. This general problem, called *graph partitioning* or *community detection*, is the subject of much recent work (see Fortunato, 2010; Fortunato and Castellano, 2009 for reviews). In this section, we apply one popular approach, modularity maximization, to the connected components of **G**, resulting in a subgraph $\mathbf{G}' \subset \mathbf{G}$. We show that by several measures for comparing graph partitions in general and rhyme graphs in particular, $\mathbf{G}'$ represents "better" data than **G**, relative to the gold standard of 1-1 correspondence between rhyme stems and components.

### 5.1. Modularity

Many community detection algorithms attempt to find a partition which maximizes a measure of partition quality. Intuitively, given a hypothesized partition of a graph into subsets, we measure how connected the vertices in each subset are to each other (relative to the rest of the graph), versus how connected we would expect them to be by chance given *no* community structure, and sum over subsets. The most commonly used formalization of this idea is *modularity*, a measure introduced by Newman and Girvan (2004).

Consider a partition $\mathcal{P}$ of a graph $G = (V, E)$ (unweighted) with $n$ vertices, $m$ edges, and adjacency matrix $A$. Consider a random graph $G' = (V, E')$, in which there are $m$ edges, vertices have the same degrees $\{a_i\}$ as in $E$, and the probability of an edge being placed between $i$ and $j$ is proportional to $a_i a_j$. The difference between the observed and expected number of edges between $i$ and $j$ is then

$$\frac{A_{ij}}{m} - \frac{a_i a_j}{2m^2}$$

This quantity is summed over all pairs of vertexes belonging to the same community:

$$Q(G, \mathcal{P}) = \sum_{\{i,j\}} \left( \frac{A_{ij}}{m} - \frac{a_i a_j}{2m^2} \right) \delta(P_i, P_j) \tag{6}$$

where $P_i$ and $P_j$ are the communities of vertices $i$ and $j$, and $\delta$ is the Kronecker delta. $Q(G, \mathcal{P})$ is called the *modularity* of $G$ under partition $\mathcal{P}$.

In the weighted case, given $G = (V, E, W)$ and partition $\mathcal{P}$, let $d_i = \sum_{i \sim j} w_{ij}$ and $m' = \sum_{\{i,j\}} w_{ij}$. By analogy to (6), modularity is defined as

$$Q(G, \mathcal{P}) = \sum_{\{i,j\}} \left( \frac{w_{ij}}{m'} - \frac{d_i d_j}{2m'^2} \right) \delta(P_i, P_j) \tag{7}$$

### 5.2. Modularity maximization

Given the weighted graph $G$ corresponding to a connected component of a rhyme graph, we would like to find the partition $\mathcal{P}^*$ which maximizes modularity: $\mathcal{P}^* = \text{argmax}_P Q(G, \mathcal{P})$. Let $Q^* \equiv Q(G, \mathcal{P}^*)$. Because the trivial partition (where $\mathcal{P} = \{G\}$) has modularity 0, $Q^* \geq 0$. It can be shown that modularity is always less than 1 (Fortunato, 2010). Thus, $Q^* \in [0, 1]$.

An exhaustive search for $\mathcal{P}^*$ intuitively seems hard due to the exponential growth of possible partitions to be checked, and is in fact NP-complete (Brandes et al., 2007). However, in practice very good approximation algorithms exist for graphs of the size considered here (Fortunato and Castellano, 2009). Such algorithms find a partition $\hat{\mathcal{P}}$ approximating $\mathcal{P}^*$, which increases modularity by $\Delta Q$. Because the trivial partition has modularity 0 and $Q^* \leq 1$:

$$0 \leq \Delta Q \leq Q^* \leq 1 \tag{8}$$

The algorithm used here is a variant of simulated annealing (SA), adapted for modularity maximization by Medus et al. (2005). Here modularity acts as the "energy" (with the difference that modularity is being maximized, while energy is usually minimized), graph vertices are "particles", transferring a vertex from one subset to another is a "transition", and the state space is the set of possible partitions. In addition, every state (partition) includes exactly one empty subset. This does not change a given partition's modularity (since no terms for the empty subset are part of the sum in Eq. (7)), but allows for transitions where a new subset (of one vertex) is created. (Whenever a vertex is transferred to the empty subset from a subset with at least 2 vertices, so that no empty subset remains, a new empty subset is added to the partition.) Our implementation of SA (in Matlab) is described in pseudocode in Algorithm 1.

**Algorithm 1.** Find a partition of $\hat{\mathcal{P}} : V \rightarrow \mathbb{N}$ of $\mathbf{G}$ which maximizes modularity (Eq. (7)), using simulated annealing.

Input: Weighted, connected $G = (V, E, W)$
$\beta \in (0, 1)$
$\alpha > 1$
`thresh`, `maxsteps`, `reps`$\in \mathbb{N}$

$Q_{\max} \leftarrow 0$
$S_{\max} \leftarrow \{V\}$
**for** $i = 1 \ldots$ `reps` **do**
  $S \leftarrow$ random initial partition of $V$ consisting of between 2 and $n-1$ subsets.
  Add an empty subset to $S$.
  $Q \leftarrow Q(W, S)$ `### from Eqn. 7`
  `t`$\leftarrow 0$, `lastaccept` $\leftarrow 0$
  **while** (`t` $-$`lastaccept`) $<$ `thresh` and `t` $<$ `maxsteps` **do**
    `t` $\leftarrow$ `t` $+ 1$
    Choose random $v \in V$.
    $s(v) \leftarrow$ subset of $S$ containing $v$.
    Choose subset $\sigma \in S \cap s(v)$.
    $S' \leftarrow S$ with $v$ moved from $s(v)$ to $\sigma$. `### proposed transition` $S \rightarrow S'$
    $Q' \leftarrow Q(W, S')$
    **if** $Q' > Q_{\max}$ **then**
      $Q_{\max} \leftarrow Q'$ `### Keep track of maximum` $Q$ `partition seen`
      $S_{\max} \leftarrow S'$
    **end if**
    $q \leftarrow \min\{1, e^{-\beta(Q-Q')}\}$.
    With probability $q$, `accept`. `### MCMC step`
    **if** `accept` **then**
      $S \leftarrow S'$
      $Q \leftarrow Q'$
      `lastaccept`$\leftarrow$`t`
      If $S$ contains no empty subset, add one.
    **end if**
    $\beta \leftarrow \alpha\beta$ `### lower pseudo-temperature`
  **end while**
**end for**
**return** $\hat{\mathcal{P}}$ corresponding to $S_{\max}$.

### 5.3. Experiment

Let $\mathcal{C}$ be the connected components of $\mathbf{G}$, with $w_{ij}$ equal to the number of times the rhyme $\{v_i, v_j\}$ occurs in the corpus. For each component $C_i \in \mathcal{C}$, we found a partition $\mathcal{P}_i = \{C_1^i, \ldots, C_{n_i}^i\}$ maximizing $Q$ by running Algorithm 1 thirty times, with $\beta = 0.01$, $\alpha = 1.01$, `lastaccept` =10000, and `maxsteps` =200000. Removing all edges between vertices in $C_j^i$ and $C_k^i$ ($\forall i; j, k \leq n_i, j \neq k$) induces a subgraph $\mathbf{G}' \subset \mathbf{G}$, with connected components $\mathcal{C}' = \bigcup_i \mathcal{P}_i$. Figs. 5 and 6 show examples of partitions found for some components of $\mathbf{G}$, discussed further below.

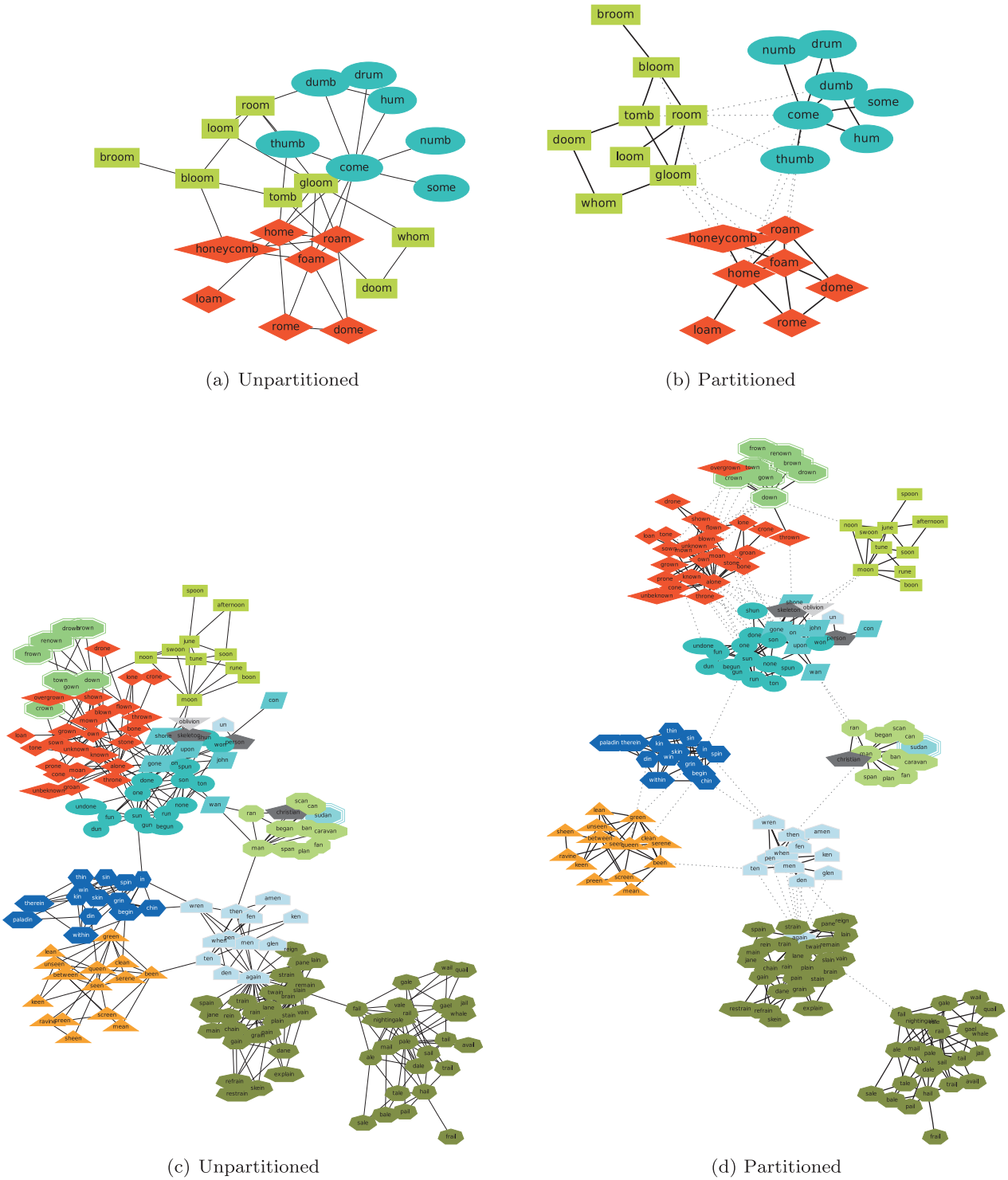
#### 5.3.1. Results

The algorithm was successful at increasing modularity by partitioning components of $\mathbf{G}$, perhaps overmuch: for *every* component $C_i \in \mathcal{C}$, a partition with higher modularity ($Q > 0$) than the trivial partition ($Q = 0$) was found. As a result, there are $|\mathcal{C}'| = 257$ components in $\mathbf{G}'$, compared with $|\mathcal{C}| = 70$ in $\mathbf{G}$. Our concern here is the extent to which these increases in modularity improve the quality of the rhyme graph. We take the "gold standard" to be a graph where there is a 1-1 correspondence between rhyme stems and components.

Does partitioning bring the rhyme graph closer to this gold standard? We first show some examples of the partitions found for individual components of $\mathbf{G}$, then discuss some quantitative measures of how the quality (made more precise below) of $\mathbf{G}'$ compares to that of $\mathbf{G}$.

### 5.4. Examples

For many components, partitioning by modularity maximization yields the desired result: a component including several rhyme stems is broken up into smaller components, each corresponding to a single rhyme stem. We give two examples.

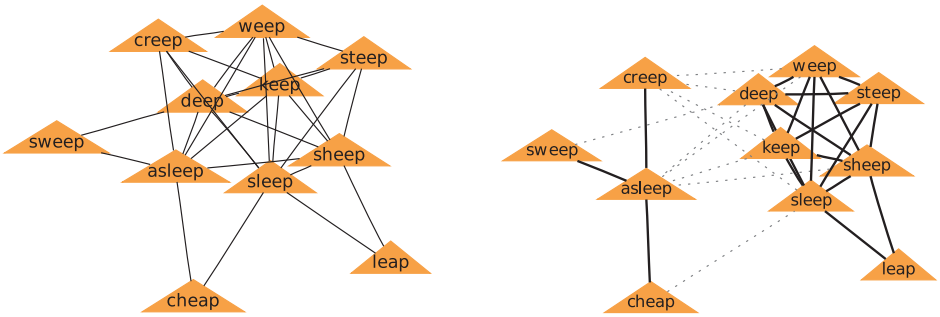



(a) Unpartitioned (b) Partitioned

(c) Unpartitioned (d) Partitioned

Fig. 5. Examples of a component of **G** corresponding to (a and b) 2 post-partitioning components of **G**′ (c and d) 10 post-partitioning components of **G**′. Dotted edges are not present in **G**′. Edge weights not shown.

1. The component of **G** shown in Fig. 5(a) corresponds to the three components of **G**′ shown in Fig. 5(b), with an increase in modularity of $\Delta Q = 0.38$. (Recall that $\Delta Q \in [0, 1]$, by Eq. (8).) The partition is also an improvement relative to the gold standard: three distinct rhyme stems ([um], [ʊm], [aʊm]) are mixed in a single component in **G**, but correspond to distinct components in **G**′.
2. The component of **G** (173 vertices) shown in Fig. 5(c) corresponds to the 10 components of **G**′ shown in Fig. 5(d), with an increase in modularity of $\Delta Q = 0.77$. The partition is a striking improvement relative to the gold standard.




(a) Unpartitioned (b) Partitioned

Fig. 6. Example of a component of **G** (a) corresponding to two post-partitioning components of **G′** (b). Dotted edges are not present in **G′**. Edge weights not shown.

Let $C$ denote the original component of **G** and $P$ the set of components of **G′**. $C$ contains 14 distinct rhyme stems ([eil], [eɪn], [in], [ɛn], [ɪn], [æn], [ɑn], [jən], [n̩], [ʌn], [ɑn], [iən], [əʊn], [un], [aʊn]). With small exceptions, each of the rhyme stems conflated in $C$ corresponds to a single component in $P$. Leaving aside the 2 rhyme stems corresponding to only one vertex each ([iən], [ɑn]), and 2 misclassified words (*overgrown*, *again*), the main errors are that $P$ splits [n̩] between two components, and puts [ʌn] and [ɑn] vertices in a single component.[16]

Recall that modularity maximization found non-trivial partitions for all components of **G**, not just those known *a priori* to contain several rhyme stems. Thus, for some components, partitioning can have a negative effect: a single positive component of **G** corresponds to multiple positive components of **G′**, as in a third example:

3. The component of **G** shown in Fig. 6(a) corresponds to the three components of **G′** shown in Fig. 6(b), with an increase in modularity of $\Delta Q = 0.07$. The effect of this partition is negative relative to the gold standard: one rhyme stem corresponds to a single component in **G**, but two components in **G′**.

For nearly all components, partitioning has one of these two effects: a component containing several rhyme stems is broken up into several components corresponding to unique rhyme stems, or a component already corresponding to a unique rhyme stem is broken up. The first kind of partition brings the rhyme graph closer to the gold standard; the second takes the rhyme graph farther from the gold standard. Importantly, a third possible kind of partition – an improvement in modularity, but a negative effect relative to the gold standard – is rarely observed.[17] Intuitively, if the effects of the first kind of partition outweigh the effects of the second kind, we expect **G′** to have higher overall quality than **G**. We now give quantitative evidence that this is the case.

### 5.5. *Measuring the quality of **G′** vs. **G***

There are many different ways the quality of a rhyme graph could be measured; we use several here. We first consider how close each of **G** and **G′** are to the gold standard, using three similarity measures from the literature for comparing clusterings of arbitrary sets. We then compare **G** and **G′** using several more intuitive measures for the particular case of rhyme graphs. For a given measure, the value for **G** measures the quality of the rhyming corpus itself; comparing to the value for **G′** measures how quality is improved by partitioning. By all measures, we find that **G′** improves significantly on **G**.

#### 5.5.1. *General measures of similarity between clusterings*

Consider a set $S = \{s_1, \ldots, s_N\}$ of $N$ data points, and let $\mathcal{U} = \{U_1, \ldots, U_R\}$ and $\mathcal{V} = \{V_1, \ldots, V_C\}$ be two partitions of $S$, into $R$ and $C$ clusters, respectively: each $s \in S$ is contained in exactly one of the $U_i$ and exactly one of the $V_j$. Let $u(s)$ and $v(s)$ be the indices of the clusters of $U$ and $V$ containing $s$: $s \in U_{u(s)}$, $s \in V_{v(s)}$. We consider two popular measures of similarity between partitions, the *adjusted Rand index* and the *normalized mutual information*, as well a

[16] We note that these are the (non-trivial) rhyme stems corresponding to the smallest numbers of vertices.

[17] For example, in a component with `MFRP =threshMFRP`, many possible partitions would give one new bad component (`MFRP<threshMFRP`) and one new good component (`MFRP>threshMFRP`).

Table 6
Measures of rhyme graph quality for **G** and **G′**, short and long rhyme stems.

| *Quality measure* | Short RS | | Long RS | |
|---|---|---|---|---|
| | **G** | **G′** | **G** | **G′** |
| vs. Gold standard | | | | |
| Adjusted rand index | 0.248 | **0.662** | 0.188 | **0.652** |
| Normalized mutual information | 0.571 | **0.637** | 0.556 | **0.634** |
| Adjusted mutual information | 0.584 | **0.777** | 0.491 | **0.674** |
| % CCs with MFRP > 0.85 | 55.7 | **79.1** | 47.1 | **68.5** |
| In CCs with MFRP > 0.85 | | | | |
| % vertices | 25.1 | **71.9** | 20.2 | **61.4** |
| % edges | 22.8 | **68.4** | 18.4 | **57.8** |
| % rhymes | 24.0 | **67.2** | 18.4 | **53.7** |
| With identical RS (all CCs) | | | | |
| % edges | 81.4 | **85.2** | 76.3 | **79.7** |
| % rhymes | 87.3 | **89.7** | 84.3 | **86.5** |

recently proposed measure, the *adjusted mutual information*. In each case, we compare the similarity between **G** and the gold standard to the similarity between **G′** and the gold standard.

*Adjusted Rand index:* Let $N_{11}$ be the number of pairs of points in $S$ which are in the same cluster in $\mathcal{U}$ and the same cluster in $\mathcal{V}$:

$$N_{11} = |\{s, t\} : s, t \in S, \quad u(s) = u(t), \quad v(s) = v(t)|$$

Define $N_{00}$ as the number of pairs which are in different clusters in $\mathcal{U}$ and different clusters in $\mathcal{V}$, $N_{01}$ as the number of pairs in different clusters in $\mathcal{U}$ but the same cluster in $\mathcal{V}$, and $N_{10}$ as the number of pairs in the same cluster in $\mathcal{U}$ but different clusters in $\mathcal{V}$. $N_{00} + N_{11}$ is then the number of pairs on which $\mathcal{U}$ and $\mathcal{V}$ agree, and $N_{01} + N_{10}$ is the number of pairs on which they disagree. The *Rand index* (Rand, 1971) is the fraction of pairs on which $\mathcal{U}$ and $\mathcal{V}$ agree:

$$RI = \frac{N_{00} + N_{11}}{N_{00} + N_{11} + N_{01} + N_{10}}.$$

However, the RI does not account for the fact that two *random* partitions will agree on many pairs. When comparing two clusterings, we would like to know whether they agree on more or fewer pairs than would be expected by chance.

Hubert and Arabie (1985) proposed adjusting the Rand index for chance, as follows. Assume that $N$ (the number of points) is fixed, and consider all pairs of partitions such that the number and size of clusters in the first and second partitions are the same as in $\mathcal{U}$ and $\mathcal{V}$, respectively. Let *Expected* and *Max* be the expected and maximum value of RI over all such partitions. The *adjusted Rand index* (ARI) is then:[18]

$$ARI = \frac{RI - Expected}{Max - Expected}$$

The ARI is 0 when $\mathcal{U}$ and $\mathcal{V}$ agree on no more pairs than expected by chance, and is bounded above by 1.

In the case of rhyme graphs, $S$ is the set of nodes (words), and $\mathcal{U}$ is the partitioning corresponding to the gold standard: each $U_i$ corresponds to a single rhyme stem, and two words $w_1, w_2 \in S$ have $u(w_1) = u(w_2)$ if their rhyme stems are identical. $\mathcal{V}$ is the partition corresponding to the rhyme graph **G** (before partitioning) or **G′** (after partitioning): each $V_j$ corresponds to a component, and $v(w_1) = v(w_2)$ if $w_1$ and $w_2$ are in the same component. For both long and short RS, we compute the ARI for each of **G** and **G′** relative to the gold standard; these are shown in Table 6. For both types of rhyme stem, ARI> 0 before partitioning, and partitioning substantially increases ARI: **G** is closer to the gold standard than would be expected by chance, but **G′** is much closer to the gold standard than **G**.

[18] *Max* and *Expected* can be written down exactly in terms of entries of the contingency table for $\mathcal{U}$ and $\mathcal{V}$. We do not do so here, for brevity.

*Normalized mutual information, adjusted mutual information:* To define the mutual information of two partitions of a set, we must first state them in terms of random variables. Let $P(i)$ denote the probability that a point $s \in S$ has $u(s) = i$, $P'(j)$ the probability that $v(s) = j$, and $P''(i, j)$ the probability that $u(s) = i$ and $v(s) = j$:

$$P(i) = \frac{|U_i|}{N}, \qquad P'(j) = \frac{|V_j|}{N}, \qquad P''(i, j) = \frac{|U_i \cap V_j|}{N}$$

The *entropy* and *mutual information* (MI) of partitions $\mathcal{U}$ and $\mathcal{V}$ are then defined via these random variables:

$$H(\mathcal{U}) = -\sum_{i=1}^{R} P(i) \log P(i), \qquad H(\mathcal{V}) = -\sum_{j=1}^{S} P'(j) \log P'(j), \qquad I(\mathcal{U}, \mathcal{V}) = \sum_{i=1}^{R}\sum_{j=1}^{S} P''(i, j) \log \left( \frac{P''(i, j)}{P(i)P'(j)} \right)$$

MI is a symmetric measure of how predictive two random variables are of each other. In the case of clustering, MI quantifies how similarly two partitions cluster the same set of data points. MI is non-negative, and is upper bounded by $\min\{H(\mathcal{U}), H(\mathcal{V})\}$. Based on these observations, Strehl and Ghosh (2003) proposed the *normalized mutual information* (NMI) as a measure for comparing partitions:

$$NMI(\mathcal{U}, \mathcal{V}) = \frac{I(\mathcal{U}, \mathcal{V})}{\sqrt{H(\mathcal{U})H(\mathcal{V})}}$$

NMI is bounded by 0 and 1. Unlike ARI, NMI is not adjusted for chance. A version of MI adjusted for chance, the *adjusted mutual information* (AMI), has recently been proposed by Vinh et al. (2009). Adjustment for chance is done similarly as for ARI, and we do not give details here. Like ARI, AMI is bounded above by 1, and is 0 when $\mathcal{U}$ and $\mathcal{V}$ agree on no more pairs than expected by chance.

For our case of rhyme graphs, we consider both NMI and AMI. $\mathcal{U}$ and $\mathcal{V}$ are defined as above (for ARI). For both long and short rhyme stems, we compare $\mathbf{G}$ and $\mathbf{G}'$ by computing NMI and AMI for each, relative to the gold standard; these are shown in Table 6. For both types of rhyme stem, partitioning increases NMI. AMI shows similar patterns to ARI: $\mathbf{G}$ is closer to the gold standard than would be expected by chance, but $\mathbf{G}'$ is significantly closer to the gold standard than $\mathbf{G}$.

### 5.5.2. Intuitive measures of rhyme graph quality

The general similarity measures just considered tell us that $\mathbf{G}'$ is closer to the gold standard than $\mathbf{G}$, but not *how* it is closer. To better understand how $\mathbf{G}$ compares with $\mathbf{G}'$, we consider several more intuitive measures of quality for the case of rhyme graphs.

*Component* `MFRP`*:* Recall that above (Section 4.2), we divided components of $\mathbf{G}$ into positive and negative classes, based on whether `MFRP` was above or below a threshold value. One measure of a graph's quality is how large the positive class is: the percentage of components with `MFRP>threshMFRP`. If we wish to weight components by their sizes, we could consider the percentage of vertices, edges (adjacent vertices) or rhymes (weights on adjacent vertices) lying in components with `MFRP>threshMFRP`.

Rows 4–7 of Table 6 give these four measures for $\mathbf{G}$ and $\mathbf{G}'$, for short and long rhyme stems. $\mathbf{G}'$ improves substantially (21–46%) on $\mathbf{G}$ in each case. Although $\mathbf{G}$ had low scores to begin with, the dramatic increases seen confirm the intuition that partitioning was largely successful in decreasing the number of components, especially large components, containing several rhyme stems.

For a more direct look at the effect of partitioning on `MFRP`, we can consider how its distribution (across all components) changed from $\mathbf{G}$ to $\mathbf{G}'$. Fig. 7 shows histograms of `MFRP` for components of $\mathbf{G}$ and $\mathbf{G}'$, for short and long rhyme stems. It is visually clear that the distributions for $\mathbf{G}'$ (partitioned) are much more concentrated near 1 (same rhyme stem for all vertices of a component) than the distributions for $\mathbf{G}$ (unpartitioned). In particular, components with `MFRP`<0.5, like the example discussed above in Fig. 5(c), are almost completely gone.

*Percentage identical rhyme stems:* Instead of measuring the size of positive components in $\mathbf{G}$ and $\mathbf{G}'$, we could consider a more basic quantity, without reference to components: the type and token percentage of full rhymes. Rows 8–9 of Table 6 show the percentage of edges between vertices with identical rhyme stems, and the analogous percentage of rhymes (again for $\mathbf{G}$, $\mathbf{G}'$, and short and long rhyme stems). $\mathbf{G}'$ again improves on $\mathbf{G}$ in all cases. Although the gains are much more modest (2–4%) than for whole components (above), they are important; they indicate that partitioning removed many more edges corresponding to half rhymes than corresponding to full rhymes, supporting the intuition

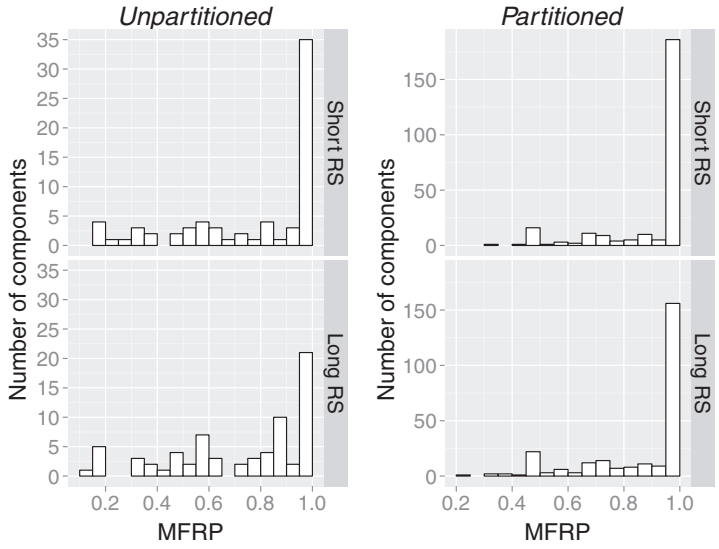



Fig. 7. Histogram of most frequent rhyme percentage (MFRP) for components of the unpartitioned (left) and partitioned (right) graphs, for short (top) and long (bottom) rhyme stems.

that many components of **G** are made up of "good" (mostly full rhymes) pieces connected by "bad" (half-rhyme) edges.

*Component cuttability:* In the classification task above, when the type of rhyme stem was fixed, we found that the distinction between positive and negative components could be boiled down to a single spectral feature, representing the quality of the best bipartition of a component: its "cuttability". For short rhyme stems, this feature was `cut lower bound 1`; for long rhyme stems it was `subset perim/area bound`. As a measure of how the cuttability of components was changed by partitioning, we can look at the distribution of these features (across components) for **G** and **G**′, shown as histograms in Figs. 8 and 9.

Recall that lower $\lambda_{11}$ and higher `subset perim/area bound` correspond to higher cuttability. $\lambda_{11}$ has mean 0.22 for components of **G** and mean 0.93 for components of **G**′; further, the distribution of $\lambda_{11}$ is much less concentrated near 0 in **G**′ than in **G**. `subset perim/area bound` has mean 1.23 for components of **G** and mean 2.06 for components of **G**′; also, its distribution is skewed right for **G** (skewness=0.84) and skewed left for **G**′ (skewness = −0.18). Overall, the distributions of $\lambda_{11}$ and `subset perim/area bound` for **G**′ versus **G** reflect that components of **G**′ are much less cuttable.

### 5.6. Summary

After partitioning **G** via modularity maximization to give **G**′, we found that by several measures, **G**′ is closer than **G** to the gold standard, where there is a 1-1 correspondence between rhyme stems and components. This improvement is

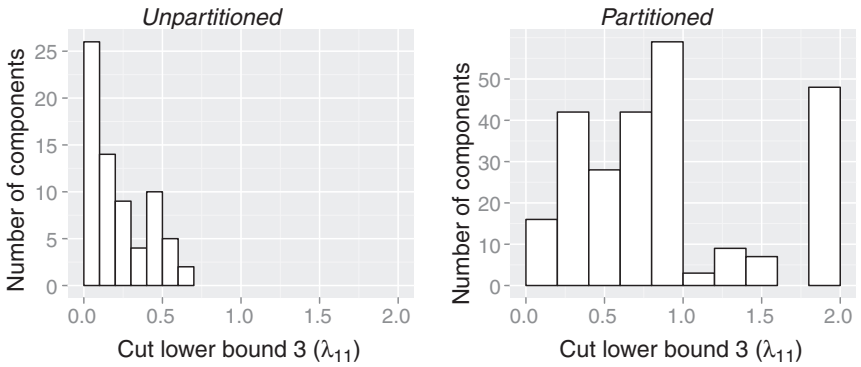



Fig. 8. Histogram of `cut lower bound 3` ($\lambda_{11}$) for components of **G** (left) and **G**′ (right).

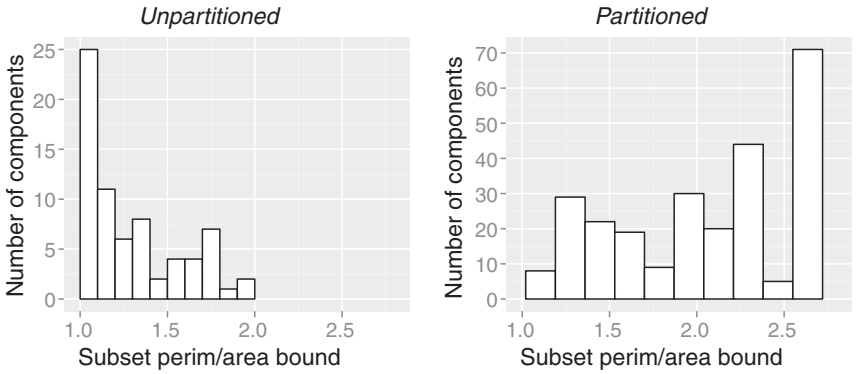



Fig. 9. Histogram of `subset perim/area bound` for components of for components of **G** (left) and **G**′ (right).

much more marked at the level of components (percentage positive components) than at the level of individual rhymes (percentage full rhymes).

## 6. Discussion

### 6.1. Future work

The methods used here are initial attempts to make use of rhyme graphs, and should be refined in future work. Allowing multiple pronunciations for a single spelling and increasing the size of the corpus should increase the quality of the data; relaxing the sanitizing steps (considering only components above a certain threshold size, excluding hapaxes) would test the robustness of our results. Different measures of components quality besides `MFRP` should be explored, as should the effect of replacing the binary classification task with a regression task (where `MFRP` is predicted from features).

*Mergers and splits:* Improvements are necessary for our long-term goal, to use the connection shown here between pronunciation and rhyme graph structure for historical inference. Suppose we had a near-perfect classifier for the "goodness" of rhyme graph components. This classifier could be applied to the graph of a historical rhyming corpus, say from poetry written around 1600. Using positive/negative labels from *current* pronunciations, we expect the classifier to make many more "errors" than for a graph corresponding to present-day data; these errors would indicate components where one of two types of pronunciation change has occurred.

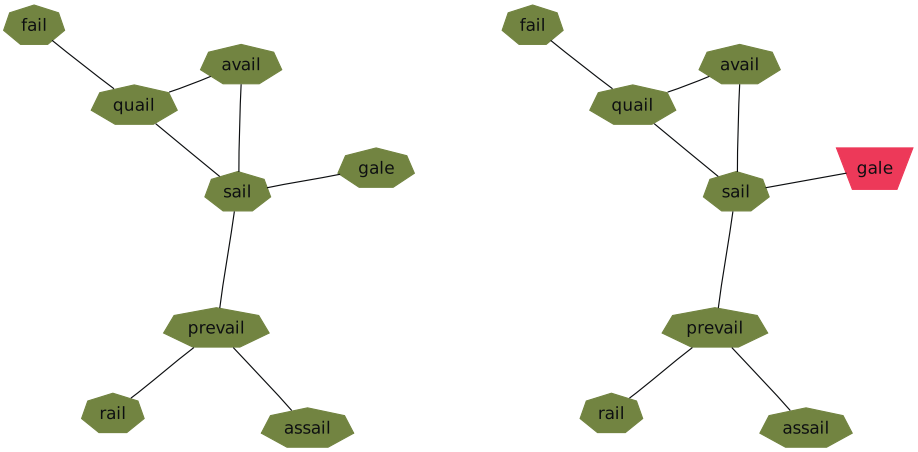
1. *Merger*: Positively labeled component classified as negative; corresponds to words whose rhyme stems were different in 1600, but are the same today.
2. *Split*: Negatively labeled component classified as positive; corresponds to words whose rhyme stems were the same in 1600, but are different today.

The trouble is that for the moment, we do not have a highly accurate classifier. Even with 90% accuracy, we cannot distinguish *a priori* between vanilla classifier errors and errors which indicate pronunciation changes.

Nonetheless, we are encouraged by some preliminary work in this direction. We constructed a corpus of poetry written around 1600, of similar size to **G**, whose graph we denote as $\mathbf{G}_{1600}$; classified its components using the most predictive feature ($\lambda_{11}$) of **G** (for short rhyme stems); and used rhyme stems from cel ex. It is indeed the case that the classifier makes many more "errors" than on **G**, and some of these errors correspond to independently known mergers and splits.

Fig. 11 shows an example split, a component of words ending (orthographically) in *-ence*. This suffix corresponds to two rhyme stems ([ɪns] and [ɛns]) in today's pronunciation (Fig. 10(a)), but a single short rhyme stem ([ɛns]) in 1600 (Fig. 10(b)).[19] Fig. 11 shows an example merger, a component of words with two rhyme stems ([ɑl], [ail]) in 1600, which have merged to a single RS ([eɪl]) today. Both examples reflect larger sound changes in English: unstressed

[19] See Footnote 2.



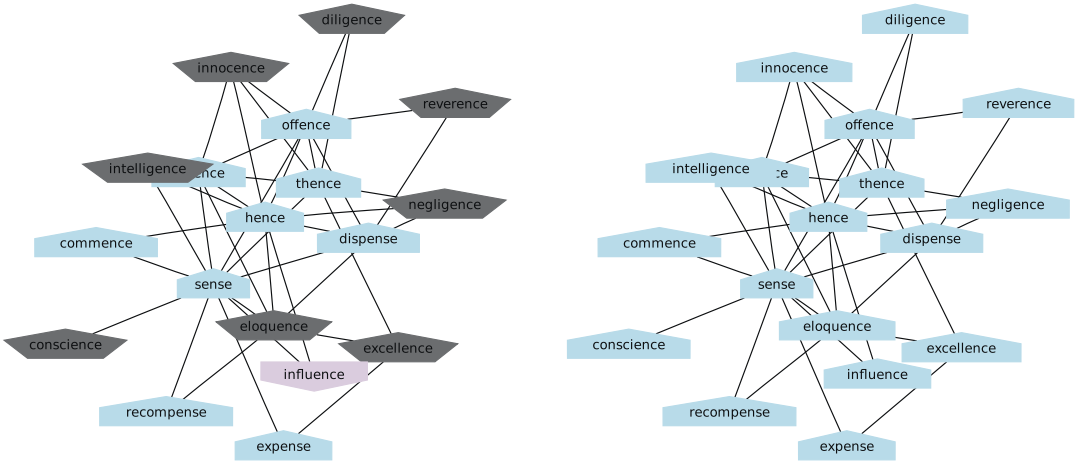

(a) 1900 pronunciation (b) 1600 pronunciation

Fig. 10. Example of a "merger": a component from $\mathbf{G}_{1600}$ classified (using short rhyme stems) as negative ($\lambda_{11} < 0.12$), with `MFRP>mfrpThresh` according to modern pronunciation (a) but `MFRP<mfrpThresh` according to 1600 pronunciations (b). Edge weights not shown.

vowels have often reduced (to [ə] or [i]) word-finally, and the vowels pronounced in Early Modern English as [ɑ] and [ai] merged to [ei] by 1800 (Lass, 1992).

*Other languages and poetic traditions:* All methods used in this paper would be straightforward to extend to rhyming corpora from other languages, provided that a pronunciation dictionary exists, and that the definition of the rhyme stem is changed appropriately. Indeed, it is important to check in future work whether the salient aspects of the English rhyme graphs considered here hold for other languages. If rhyme graphs do not show some sort of similar structure cross-linguistically, they cannot be used for pronunciation reconstruction in the most interesting cases, where the historical pronunciation of a language is *unknown*.

The methods used in this paper are also applicable to data from other poetic traditions. Rhyming in modern English poetry requires that pairs of words be similar in a particular way near their endings. Different poetic traditions require that sets of words be similar, but define similarity very differently. In *alliterative verse*, pairs of words must begin with the same phonemes; this is the dominant structuring device in most verse written in Old English (such as *Beowulf*) and Old Norse (the ancestor of modern-day Icelandic) (Godden and Lapidge, 1991; Ross, 2005). In Welsh poetry, different types of *cynghanedd* ("harmony") require various complex patterns of consonantal correspondence and rhyming among words within individual lines (Williams, 1952; Turco, 2000). In principle, the methods used in this paper could be




(a) 1900 pronunciation (b) 1600 pronunciation

Fig. 11. Example of a "split": a component from $\mathbf{G}_{1600}$ classified (using short rhyme stems) as positive ($\lambda_{11} > 0.12$), with `MFRP<mfrpThresh` according to modern pronunciation (a) but `MFRP>mfrpThresh` according to 1600 pronunciations (b). Edge weights not shown.

extended to data from any poetic tradition, like alliterative verse or *cynghanedd*, where some sort of similarity between the pronunciation of sets of words is implied by poetic form.

*Full rhyme to half rhyme ratio* For the poetic corpus considered here – English rhyming verse written around 1900 – we found that the rhyme graph largely consists of full-rhyming clusters, connected by half-rhyming edges. A natural extension would be to check how robust this finding is for rhyming corpora where the ratio of half rhymes to full rhymes is greater. In general, verse from various genres and dates will differ in how common half rhymes are relative to full rhymes. For example, half rhymes seem to be more frequent in (contemporary, English-language) song lyrics than in rhyming poetry: in Katz' (2008) English hip-hop corpus, 56% of rhymes have identical long rhyme stems, compared to 84% in our corpus. Half rhymes also may be more common in translations of rhyming verse into English, where faithfulness to the rhyme scheme may require that the translator use more half rhymes.

### 6.2. Summary

In Sections 2 and 3, we introduced a corpus of rhymes from recent poetry, and explored its rhyme graph, **G**. We found most components of **G** either consist of a densely connected set of vertices (with edges corresponding to full rhymes), or several such sets, with few edges between sets (corresponding to half rhymes); relatively few components correspond to spelling rhymes or poetic pronunciation conventions. In other words, graph structure for the most part transparently reflects actual pronunciation. This is not a trivial fact: it could have been the case that half rhymes occur frequently enough to obscure the distinction between half and full rhymes, or that spelling rhymes or poetic pronunciation conventions are widespread. That structure reflects pronunciation in poetry means it is (in principle) possible to "read off" pronunciation from structure, as discussed above.

In Section 4, we found that spectral features are much more predictive of component "goodness" than non-spectral features. Though it possible that a different set of non-spectral features would perform better, it is striking that for both short and long rhyme stems, *no* non-spectral feature is more predictive than *any* spectral feature. We tentatively conclude that a component's eigenvalue spectrum is more predictive of its "goodness" (i.e. class label) than the non-spectral measures often used in network research. Overall, we confirmed the intuition that component goodness corresponds, for the most part, to whether a good partition exists.

In Section 5, we found that applying modularity-based partitioning to components of **G**, resulting in a new graph $\mathbf{G}'$, significantly improves the quality of the data, especially when seen from the perspective of components, rather than individual rhymes. For the short RS case, for example, 79% of components in $\mathbf{G}'$ are positive, corresponding to 72% of words, compared to 56%/25% for **G**. For the long-term goal of using rhyme graphs for pronunciation reconstruction, this is our most important finding: by partitioning components, we go from 50/50 positive/negative components to 80/20. Where a random component of **G** contains many half rhymes at chance, a random component of $\mathbf{G}'$ probably does not.

Overall, we have shown three cases where it is possible and profitable to understand groups of rhymes in terms of their corresponding rhyme graphs. We can roughly summarize our findings by three correspondences between a given group of rhymes $R$, corresponding to a connected component $\mathbf{G}(R)$ of rhyme graph **G**:

| *Group of rhymes* | | *Component of rhyme graph* |
|---|---|---|
| Most rhymes in $R$ are full, fewer are half. | $\Leftrightarrow$ | $\mathbf{G}(R)$ has community structure. |
| $R$ contains half-rhymes. | $\Leftrightarrow$ | $\mathbf{G}(R)$ has a good partition. |
| Which groups of rhymes in $R$ are definitely full? | $\Leftrightarrow$ | What is the best partition of $\mathbf{G}(R)$? |

We thus add to the recent body of work illustrating that in very different settings (e.g. Luce and Pisoni, 1998; Ferrer i Cancho and Sole, 2001; Sigman and Cecchi, 2002; Steyvers and Tenenbaum, 2005; Mukherjee et al., 2008; Vitevitch, 2008; Mukherjee et al., 2009a; Arbesman et al., 2010; Ferrer i Cancho, 2010 gives a bibliography), considering linguistic data as graphs (or networks) gives new insights into how language is structured and used. Specifically, like (Ferrer i Cancho et al., 2007; Biemann et al., 2009; Mukherjee et al., 2009b), we found a strong and striking association between graph spectra and linguistic properties.

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