Distributing the disjuncts over the modal space *

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1. The distribution requirement

Time for dessert. Nobody can have anything unless Mom has explicitly given her the option to. And nobody goes against Mom. Those are the rules and this is how the game is played:

(1) a. Dad, to Mom: "For dessert, we have cake, ice cream, and *crème caramel*."
b. Mom, to Leonor: "You may have *crème caramel*."
c. Mom, to Sandy: "You may have either this piece of cake or that ice cream."

What has Leonor learned about her permitted dessert options? That she has one: having *crème caramel*. Were she to have cake, she would be disobedient — given the rules of the game, if Mom had wanted her to have cake, she would have surely said so. What has Sandy learned about her dessert options? At least three things: (i) that having cake *and* ice cream is out of question — what l will call the *exclusivity requirement* — (ii) that having *crème caramel* is not a permitted option (if Mom had wanted her to have *crème caramel*, she would have said so) — what I will call the *exhaustivity requirement* — and (iii) that she has *two* dessert options and *each* is permitted — the *distribution requirement* (a term borrowed from Kratzer and Shimoyama (2002)). For the most part, only the distribution requirement will concern us here.

The problem is an old one: the standard semantics of modals and *or* does not deliver the distribution requirement (von Wright, 1968; Kamp, 1973). To see why, take the sentence in (1c). I assume that *may* is a raising predicate, that its surface structure subject is not its own argument and reconstructs back under its scope at LF, as in (2a). The textbook semantics of deontic *may* requires that the proposition expressed by its sister constituent be compatible with the set of permitted worlds.

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(2) a. $[_{CP}$ may $[_{IP}$ you have [either this piece of cake or that ice cream]]]

b. Where $[\![\alpha]\!] \in D_{\langle s,t\rangle}$ and $\mathscr{D}_w$ is the set of worlds deontically accessible from w, $[\![\text{may}[\alpha]]\!](w) = 1 \Leftrightarrow \exists w' : w' \in \mathscr{D}_w.[\![\alpha]\!](w') = 1$

The textbook semantics for *or* assumes that it denotes the boolean join and assigns to the IP in (2a) the proposition defined by the expression in (3):[1]

(3) $\lambda w.\mathbf{have}_w(\mathbf{sandy}, \mathbf{this\text{-}cake}) \vee \mathbf{have}_w(\mathbf{sandy}, \mathbf{that\text{-}ice\text{-}cream})$

The proposition defined by the expression in (3) is true of worlds in which Sandy has this piece of cake, she has that ice cream, or she has both. All the semantics in (2b) requires is that this proposition be compatible with the set of permitted worlds. The proposition in (3) can be compatible with the set of permitted worlds if there is no permitted world where Sandy has this piece of cake (as long as she is permitted to have this ice cream), or in case she is not permitted to have that ice cream (as long as she is permitted to have this piece of cake). Nothing requires that the dessert options be *distributed* among the permitted worlds — that there be permitted worlds where Sandy has this piece of cake and that there be permitted worlds where Sandy has that ice cream. That is the central issue of this paper.

The same happens with necessity modals. Consider, as illustration, a second scenario. Every month Dad, Sandy and Leonor spend a full Sunday doing home chores. Dad decides what to do and who must do what. Nobody is required to do anything unless Dad explicitly says so. This is what happened last Sunday:

(4) a. Dad, to Leonor and Sandy: “We have to clean some bedrooms, paint the barn and mow the lawn.”

b. Dad, to Leonor: “You must clean your bedroom.”

c. Dad, to Sandy: “You must either clean your bedroom or mow the lawn.”

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[1] I assume that disjunctions are always *balanced* (that its disjuncts are always of the same category) and that *either* marks the left edge of the disjunction (Schwarz, 1999). *Or* disjoins two DPs in (1c), then. According to the boolean-join hypothesis, *or* maps the denotation of its disjuncts to their join. The cross-categorial definition of this operation, which relies on the recursive definition of conjoinable types in (i), is given below (Partee and Rooth, 1983).

(i) a. t is a conjoinable type

b. if τ is a conjoinable type, $\langle\sigma,\tau\rangle$ is a conjoinable type, for any σ.

(ii) For any α, β of conjoinable type τ, $[\![\alpha \text{ or } \beta]\!] = [\![\alpha]\!] \sqcup [\![\beta]\!]$

a. for $T_1, T_2 \in D_t, T_1 \sqcup T_2 = T_1 \vee T_2$ (= True iff T_1 = True or T_2 = True)

b. for $f_1, f_2 \in D_{\langle\sigma,\tau\rangle}, f_1 \sqcup f_2 = \lambda s_\sigma. f_1(s) \sqcup f_2(s)$

The disjunction in (1c) is the join of two generalized quantifiers in (iii) below, which applied to (iv) returns the proposition defined by the expression in (3).

(iii) $[\![\, [_{\,DP} [_{DP_1}$ this cake] or $[_{DP_2}$ that ice cream]] $]\!] = \lambda P.\lambda w.P_w(\mathbf{this\text{-}cake}) \sqcup P_w(\mathbf{that\text{-}ice\text{-}cream})$

(iv) $[\![\, [1$ [you have t_1]] $]\!] = \lambda x.\lambda w.\mathbf{have}_w(\mathbf{you}, x)$

What has Leonor learned about her home chores obligations? That she has one: cleaning her bedroom. She can probably conclude that she is not required to paint the barn and that she is not required to mow the lawn — for otherwise Dad would have said so. What has Sandy learned? At least two things: (i) that not doing any of those two chores is not permitted and (ii) that both are permitted: she may clean her bedroom and she may mow the lawn. The sentence in (4c) also triggers the distribution requirement. The textbook semantics, however, fails to deliver it here too. For assume that the structure to be interpreted is as in (5a). The textbook semantics of deontic *must* makes it the dual of *may*:

(5) a. $[_{\mathrm{CP}}$ must $[_{\mathrm{IP}}$ you [clean your bedroom or mow the lawn]]]

b. Where $[\![\alpha]\!] \in D_{\langle s,t\rangle}$ and $\mathscr{D}_w$ is the set of worlds deontically accessible from w, $[\![\text{must}[\alpha]]\!](w) = 1 \Leftrightarrow \forall w' : w' \in \mathscr{D}_w.[\![\alpha]\!](w') = 1$

The proposition expressed by the sister constituent of *must* is the one defined by the expression in (6):

(6) $\lambda w.\mathbf{clean}_w(\mathbf{sandy}, \mathbf{your\text{-}bedroom}) \vee \mathbf{mow}_w(\mathbf{sandy}, \mathbf{the\text{-}lawn})$

This proposition can be true in all permitted worlds if Sandy is not permitted to clean her bedroom (as long as she is required to mow the lawn), or if she is not permitted to mow the lawn (as long as she is required to clean her bedroom). Nothing makes sure that cleaning her bedroom and mowing the lawn are both permitted.

2. Two analyses

The textbook semantics doesn't deliver the distribution requirement. How can we obtain it? There are two types of answers. The first, which I will refer to as *Analysis 1*, has it that the distribution requirement is truth-conditional. Since the textbook semantics does not deliver this entailment, Analysis 1 proposes an alternative semantics for disjunction (Zimmerman, 2001; Geurts, 2004) or for both modals and disjunction (Aloni, 2002; Simons, 2005). A second strategy derives the distribution requirement as a conversational implicature. The strategy has been entertained under different guises in Blutner (2003); Schulz (2004) and Kratzer and Shimoyama (2002) (for free choice indefinites). I will ignore, for reasons of space, the first two variants and focus on an implementation of the Kratzer and Shimoyama (2002) analysis, which I will refer to as *Analysis 2*.[2]

My modest goal is to present two problems for Analysis 1 that Analysis 2 circumvents. The first, which applies to all variants of Analysis 1, is that it fails to assign the right truth conditions for disjunctions in downward-entailing environments. The second, which applies more directly to the compositional versions in Aloni (2002) and Simons (2005), is that it predicts unattested intervention effects.

[2]Vainikka (1987) derives the distribution requirement from a presupposition of *or*. I can't discuss her analysis for reasons of space. It also faces the problem in downward entailing expressions (Alonso-Ovalle, 2004).

3. Analysis 1: the distribution requirement is truth-conditional

Analysis 1 imports the distribution requirement into the truth-conditions. The sentence in (7), as uttered by Mom, is assumed to *assert* that both options (having cake and having ice cream) are permitted. Its content can be then paraphrased as in (8).

(7) Sandy may have either this piece of cake or that ice cream.

(8) $$\forall p \in \left\{ \begin{array}{c} \lambda w.\mathbf{has}_w(\mathbf{sandy}, \mathbf{this\text{-}cake}), \\ \lambda w.\mathbf{has}_w(\mathbf{sandy}, \mathbf{that\text{-}ice\text{-}cream}) \end{array} \right\} (\Diamond p)$$

The varieties of Analysis 1 differ as to how to get to the meaning in (8). In Zimmerman 2001, *or* provides a conjunction of epistemic possibilities. The sentence in (7) asserts that it is epistemically possible that Sandy may have this cake and also that she may have that ice cream:[3]

(9) $$\nabla(\Diamond(\lambda w.\mathbf{has}_w(\mathbf{sandy}, \mathbf{this\text{-}cake}))) \;\&\; \nabla(\Diamond(\lambda w.\mathbf{has}_w(\mathbf{sandy}, \mathbf{that\text{-}ice\text{-}cream})))$$

By assuming that the speaker is infallible about what Sandy may or may not do (that if, according to the speaker, it might be true that she may eat this ice cream, Sandy in fact may eat this ice cream), (9) entails the conjunction of deontic possibilities in (10), the analysis that Geurts (2004) proposes for (7).

(10) $$\Diamond(\lambda w.\mathbf{has}_w(\mathbf{sandy}, \mathbf{this\text{-}cake})) \;\&\; \Diamond(\lambda w.\mathbf{has}_w(\mathbf{sandy}, \mathbf{that\text{-}ice\text{-}cream}))$$

Aloni (2002) and Simons (2005) offer a compositional version of the analysis. The paraphrasis in (8) has two components: (a) a domain of propositions and (b) a universal quantifier that distributes them over the space of permitted options; under their analysis, *or* introduces the set of propositions and *may* imposes the distribution.

Commands cannot be given a parallel analysis. The sentence in (11a), as uttered by Dad, doesn't imply that both options are *obligatory*, but rather that both are *permitted* (and that not doing either of the two is not permitted) as in (11b).

(11) a. You must either clean your bedroom or mow the lawn.

b. $$\forall p \in \left\{ \begin{array}{c} \lambda w.\mathbf{clean}_w(\mathbf{sandy}, \mathbf{sandy's\text{-}bedroom}), \\ \lambda w.\mathbf{mow}_w(\mathbf{sandy}, \mathbf{the\text{-}lawn}) \end{array} \right\} (\Diamond p) \;\& \\ \Box(\lambda w.\mathbf{clean}_w(\mathbf{sandy}, \mathbf{sandy's\text{-}bedroom}) \vee \mathbf{mow}_w(\mathbf{sandy}, \mathbf{the\text{-}lawn}))$$

Only Geurts (2004) and Simons (2005) derive the distribution requirement with *must*.[4] Geurts (2004) analyzes (11a) as a conjunction of necessity statements whose domains are required to be disjoint and to exhaust together the space of deontic possibilities:[5]

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[3]Notation: "∇" stands for epistemic possibility and "$\Diamond$" for deontic posibility.

[4]Zimmerman (2001) acknowledges that his analysis doesn't derive the distribution requirement for *must*. Aloni (2002) treats *must* as an existential quantifier over propositional alternatives to derive the ungrammaticality of *any* under the scope of *must*. That move does not derive the distribution requirement.

[5]Notation: the subscripts in the necessity operators are meant to indicate the domain of quantification.

(12) $\Box_{\mathscr{D}_1}(\lambda w.\mathbf{clean}_w(\mathbf{sandy}, \mathbf{the\text{-}bedroom}))\ \&\ \Box_{\mathscr{D}_2}(\lambda w.\mathbf{mow}_w(\mathbf{sandy}, \mathbf{the\text{-}lawn}))$
where $\mathscr{D}_1 \cup \mathscr{D}_2 = \mathscr{D}_w$ and $\mathscr{D}_1 \cap \mathscr{D}_2 = \emptyset$

In the compositional version in Simons (2005), *or* only sets up a domain of propositional alternatives, over which the distribution requirement is imposed when combined with *must*.

3.1. Downward-entailing environments

Consider a variant of the situation in (1) in which this is the only rule:

(13) Mom, to Sandy and Leonor: "None of you may have this cake or that ice cream."

Given what Mom wants, if Sandy had ice cream, she would be disobedient. Analysis 1 does not predict so. The sentence in (13) cannot be interpreted as in (14), where the negative quantifier has wide scope. If it were, it would be true if Sandy and Leonor were both allowed to have cake (but not ice cream).

(14) $$\neg\exists x : \mathbf{child}(x) \left(\forall p \in \left\{ \begin{array}{c} \lambda w.\mathbf{has}_w(x, \mathbf{this\text{-}cake}), \\ \lambda w.\mathbf{has}_w(x, \mathbf{that\text{-}ice\text{-}cream}) \end{array} \right\} (\Diamond p) \right)$$

It cannot be interpreted as (15), either. If it were, it would be true if both Sandy and Leonor were allowed to have ice cream (but not cake) or if they were allowed to have both.

(15) $$\forall p \in \left\{ \begin{array}{c} \lambda w.\neg\exists x(\mathbf{child}(x)\ \&\ \mathbf{has}_w(x, \mathbf{this\text{-}cake})), \\ \lambda w.\neg\exists x(\mathbf{child}(x)\ \&\ \mathbf{has}_w(x, \mathbf{that\text{-}ice\text{-}cream})) \end{array} \right\} (\Diamond p)$$

Likewise for the second scenario. Suppose Dad had ruled as in (16):

(16) Dad: "None of you must clean the bedroom or mow the lawn."

Given what he wants, Sandy is not required to clean her bedroom — nor to mow the lawn. The meaning of (16) cannot be paraphrased as in (17). If it were, the sentence in (16) would be true if both Sandy and Leonor had to mow the lawn (as long as they wouldn't be allowed to clean their bedroom).

(17) $$\neg\exists x : \mathbf{child}(x) \left(\begin{array}{c} \forall p \in \left\{ \begin{array}{c} \lambda w.\mathbf{has}_w(x, \mathbf{this\text{-}cake}), \\ \lambda w.\mathbf{has}_w(x, \mathbf{that\text{-}ice\text{-}cream}) \end{array} \right\} (\Diamond p) \\ \& \\ \Box(\lambda w.\mathbf{has}_w(x, \mathbf{this\text{-}cake}) \vee \mathbf{has}_w(x, \mathbf{that\text{-}ice\text{-}cream})) \end{array} \right)$$

This is the first challenge for Analysis 1. A second, for its compositional versions, follows.

3.2. Unattested intervention effects

In the compositional versions of Analysis 1 (Aloni, 2002; Simons, 2005), the distribution requirement is imposed over a set of propositional alternatives, introduced by *or*:

(18) a. Sandy may have this piece of cake or that ice cream.

b. $\forall p \in \left\{ \begin{array}{c} \lambda w.\mathbf{has}_w(\mathbf{sandy}, \mathbf{this\text{-}cake}), \\ \lambda w.\mathbf{has}_w(\mathbf{sandy}, \mathbf{that\text{-}ice\text{-}cream}) \end{array} \right\} (\Diamond p)$

For the disjuncts to be distributed, their semantic identity must be preserved. Under the boolean-join hypothesis, however, the semantic identity of the disjuncts is destroyed. The inverse of join formation is not a function. Once the disjuncts are mapped into their join, they cannot be retrieved. To the extent that join formation applies to the set of propositional alternatives created by *or*, as in (19), the disjuncts are rendered invisible and the distribution requirement is predicted to be blocked.

(19) $\forall p \in \{\lambda w.\mathbf{has}_w(\mathbf{sandy}, \mathbf{this\text{-}cake}) \vee \mathbf{has}_w(\mathbf{sandy}, \mathbf{that\text{-}ice\text{-}cream})\}(\Diamond p)$

The hypothesis that the disjuncts remain accessible for external operators was first defended in recent times by Rooth and Partee (1982). For them, *or* does not have any quantificational force of its own: it simply restricts the possible values of a free variable over which other external operators, like the implicit universal in (20b), range.

(20) a. If Mary is swimming or dancing, then Sue is. (Rooth and Partee, 1982)

b. $\forall w, P(P_w(\mathbf{mary})\ \&\ (P = \mathbf{swimming} \vee P = \mathbf{dancing})) \rightarrow P_w(\mathbf{sue}))$

Two arguments suggest that *either. . . or* disjunctions contrast with *or*-disjunctions in that they have existential force and, hence, their disjuncts are rendered invisible to external operators. Larson (1985) presents the first. He contrasts (20a) with (21).

(21) If Mary is either swimming or dancing, then Sue is. (Larson, 1985)

Rooth and Partee's example has one reading, captured in (20b), under which Sue is required to be doing whatever Mary is. Larson notes that his example does not. This, as he notes, would follow as an intervention effect if *either* marked Existential Closure, in which case the variable introduced by disjunction couldn't be bound by the implicit universal.

(22) $\forall w \left(\begin{array}{c} \exists P(P_w(\mathbf{mary})\ \&\ (P = \mathbf{swimming} \vee P = \mathbf{dancing})) \\ \rightarrow \\ \exists P(P_w(\mathbf{sue})\ \&\ (P = \mathbf{swimming} \vee P = \mathbf{dancing})) \end{array} \right)$

Questions provide a second argument. *Either. . . or* disjunctions are ruled out in alternative questions:

(23) a. Mom: “Did Sandy have either cake or ice cream?”

b. Dad: “Yes.” (= She had at least one of the two.) / “No.” (= She didn't.)

c. Dad: # “Cake.” / # “Ice cream.”

This can be yet another intervention effect. Assume the Rooth & Partee analysis. Alternative questions can be derived as Berman (1991) derives constituent questions: by letting a question forming operator bind the variable introduced by *or*.

(24) a. Which students came?
$[\![?_1[\text{student}]_1[\text{t}_1 \text{ came}]]\!]^{w,g} =$
$\lambda p.\exists x(\mathbf{student}_w(x) \;\&\; p = \lambda w'.\mathbf{came}_{w'}(x))$

b. Did Sandy have this cake, or that ice cream?
$[\![?_1[\text{this cake or that ice cream}]_1[\text{Sandy had } t_1]]\!]^{w,g} =$
$\lambda p.\exists x((x = \textbf{this-cake} \vee x = \textbf{that-ice-cream}) \;\&\; p = \lambda w'.\mathbf{had}_{w'}(\mathbf{sandy}, x))$

If *either... or* disjunctions are existentially closed, the question forming operator cannot bind the variable they introduce. The alternative question reading is then blocked.

(25) Did Sandy have either this cake or that ice cream?
$?_1\exists_1[\text{this cake or that ice cream}]_1[\text{Sandy had } t_1]$

If *either... or* disjunctions make the disjuncts inaccessible for the implicit universal in conditionals and for the question forming operator, they should also make the disjuncts inaccessible for modals. But if modals do not have access to the disjuncts, under the setup in Aloni 2002 and Simons 2004, the distribution requirement is predicted to be blocked.

(26) a. Sandy may have either this cake or that ice cream.

b. $$\forall p \in \left\{ \lambda w.\exists x \left(\left(\begin{array}{c} x = \textbf{this-cake} \\ \vee \\ x = \textbf{that-ice-cream} \end{array} \right) \;\&\; \mathbf{has}_w(\mathbf{sandy}, x) \right) \right\} (\Diamond p)$$

This, as example (1) illustrates, is wrong.

3.3. Summing up

This is what we have learned so far: (i) the distribution requirement is absent in downward-entailing environments, and (ii) it can be triggered even if *or* is existentially closed. Being unnoticeable in downward entailing environments is the hallmark of quantity-based implicatures. Enter Analysis 2.

4. Analysis 2: The implicatures of domain widening

Analysis 2 derives the distribution requirement as an implicature of domain widening *à la* Kratzer and Shimoyama (2002), one that is drawn when the propositional alternatives created by *or* are caught by the Existential Closure operator, routinely triggered in the scope of modals.

A Hamblin semantics is adopted. Expressions of type τ are mapped into *sets* of objects in D_τ. Following Rooth and Partee (1982), I assume that *or* lacks any quantificational force. Its only role is to collect the denotation of its disjuncts in a set (as is also proposed in Simons 2005).

(27) Where $[\![\mathrm{A}]\!], [\![\mathrm{B}]\!] \subseteq \mathrm{D}_\tau$, $\left[\!\!\left[\; [_{\mathrm{orP}} \; A \;\; or \;\; B \,] \; \right]\!\!\right] \subseteq \mathrm{D}_\tau = [\![\mathrm{A}]\!] \cup [\![\mathrm{B}]\!]$

Take, as an illustration, the DP disjunction below:

(28) $[_{\mathrm{DP}}$ $[_{\mathrm{DP}_1}$ this cake] or $[_{\mathrm{DP}_2}$ that ice cream]]

Each disjunct can now denote a singleton containing an individual. *Or* collects each individual in a set. The denotation of the disjunction contains *two* distinct elements: the semantic identity of the disjuncts is retained. Likewise for the VP disjunction in (bb).

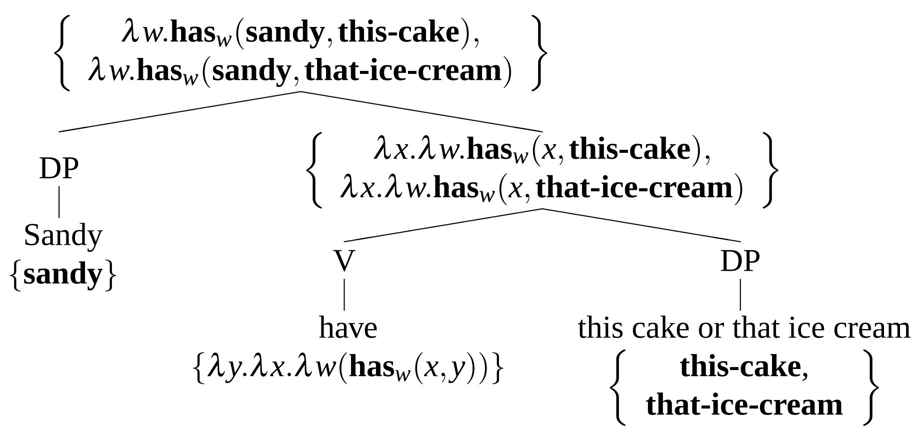
(29) a. $[\![$[$[_{\mathrm{DP}_1}$ this cake] or $[_{\mathrm{DP}_2}$ that ice cream]]$]\!]$ = $\{$**this-cake**, **that-ice-cream**$\}$

b. $[\![$[$[_{\mathrm{VP}_1}$ clean the bedroom] or $[_{\mathrm{VP}_2}$ mow the lawn]]$]\!]$ =
$\{\lambda x.\lambda w.\mathbf{clean}_w(x, \mathbf{the\text{-}bedroom}), \lambda x.\lambda w.\mathbf{mow\text{-}the\text{-}lawn}_w(x)\}$

The alternatives introduced by *or* combine by pointwise functional application (30) and grow propositional (as in Simons (2005)). The process is illustrated in (31):

(30) Where $[\![\alpha]\!] \subseteq D_{\langle\sigma,\tau\rangle}$ and $[\![\beta]\!] \subseteq D_\sigma$,
$[\![\alpha(\beta)]\!] = \{c \in D_\tau : \exists a \in [\![\alpha]\!] \exists b \in [\![\beta]\!] (c = a(b))\}$ (Hamblin, 1973)

(31)



Unlike in Aloni 2002 and Simons 2005, modals do not have access to the set of alternatives. The set of propositional alternatives created by *or* is caught by Existential Closure (in (32)), which is usually assumed to be triggered under the scope of modals (Heim, 1982).

(32) Where $[\![\mathbb{A}]\!] \subseteq D_{\langle s,t\rangle}$,
$[\![\exists(\mathbb{A})]\!] = \{\lambda w.\exists p \in [\![\mathbb{A}]\!] \;\&\; p(w) = 1\}$ (Kratzer and Shimoyama, 2002)

Unlike in Aloni (2002), the denotation of modals is standard — although they are defined for sets of propositions. *May* is an existential over worlds and *must* is a universal.

(33) $[\![\mathrm{may}(\mathbb{A})]\!] = \;\{\lambda w.\exists w' : w' \in \mathscr{D}_w.\ \exists p \in [\![\mathbb{A}]\!] \;\&\; p(w') = 1\}$

(34) $[\![\text{must } (\mathbb{A})]\!] = \{\lambda w. \forall w' : w' \in \mathscr{D}_w.\ \exists p \in [\![\mathbb{A}]\!]\ \&\ p(w') = 1\}$

Under this setup, the role of *or* is to specify the domain of Existential Closure. Adding disjuncts — using a disjunction, to begin with — amounts to widening the domain of Existential Closure:

(35) $[\![\text{may } \exists(\text{Sandy has this cake})]\!] =$
$\{\lambda w. \exists w' : w' \in \mathscr{D}_w. \exists p \in \{\lambda w. \mathbf{has}_w(\mathbf{sandy}, \mathbf{this\text{-}cake})\}\ \&\ p(w') = 1\}$

(36) $[\![\text{may } \exists(\text{Sandy has this cake or that ice cream})]\!] =$
$$\left\{\lambda w. \exists w' : w' \in \mathscr{D}_w. \exists p \in \left\{\begin{array}{c} \lambda w. \mathbf{has}_w(\mathbf{sandy}, \mathbf{this\text{-}cake}), \\ \lambda w. \mathbf{has}_w(\mathbf{sandy}, \mathbf{that\text{-}ice\text{-}cream}) \end{array}\right\}\ \&\ p(w') = 1\right\}$$

(37) $[\![\text{may } \exists(\text{Sandy has this cake or that ice cream or that pear})]\!] =$
$$\left\{\lambda w. \forall w' : w' \in \mathscr{D}_w. \exists p \in \left\{\begin{array}{c} \lambda w. \mathbf{has}_w(\mathbf{sandy}, \mathbf{this\text{-}cake}), \\ \lambda w. \mathbf{has}_w(\mathbf{sandy}, \mathbf{that\text{-}ice\text{-}cream}), \\ \lambda w. \mathbf{has}_w(\mathbf{sandy}, \mathbf{that\text{-}pear}) \end{array}\right\}\ \&\ p(w') = 1\right\}$$

Existential Closure is upward entailing. If (35) is true, so will (36) and (37) be. Not the other way around, though. In positive environments, then, using a disjunction (with a certain number of terms) weakens the assertion. That move goes against the maxim of quantity. It has to be justified. This is where Kratzer and Shimoyama (2002) enter the stage. Consider again the dialogue in (1):

(38) a. Dad, to Mom: “For dessert, we have cake, ice cream, and *crème caramel*.”

b. Mom, to Leonor: “You may have *crème caramel*.”

c. Mom, to Sandy: “You may have either this piece of cake or that ice cream.”

Upon hearing (38b), Leonor concluded that she may not have ice cream —remember that, given the rules, had Mom wanted Leonor to have ice cream, she would have surely said so. Now consider her utterance in (38c). Mom chose this time a bigger domain for Existential Closure, one containing two propositions. She could have chosen a smaller domain and uttered either of the stronger sentences in (39a-39b).

(39) a. Sandy may have this piece of cake.
$[\![\text{may}(\exists(\text{Sandy has this piece of cake}))]\!] =$
$\{\lambda w. \exists w' : w' \in \mathscr{D}_w. \exists p \in \{\lambda w. \mathbf{has}_w(\mathbf{sandy}, \mathbf{this\text{-}cake})\}\ \&\ p(w') = 1\}$

b. Sandy may have that ice cream.
$[\![\text{may}(\exists(\text{Sandy has that ice cream}))]\!] =$
$\{\lambda w. \exists w' : w' \in \mathscr{D}_w. \exists p \in \{\lambda w. \mathbf{has}_w(\mathbf{sandy}, \mathbf{that\text{-}ice\text{-}cream})\}\ \&\ p(w') = 1\}$

We can wonder, in the way familiar from the derivation of quantity implicatures, why she didn’t do so. Given what Leonor has concluded, uttering (39a) could have led Sandy to conclude that she is not allowed to have ice cream. By widening the domain of Existential Closure, Mom signals that the exhaustivity inference that (39b) is false is to be avoided.

We conclude from (38c) that if (39a) is true, then so is (39b). By parity of reasoning, we can conclude that if (39b) is true, then so is (39a).[6]

(40) (37c) $+>$ ($[\![(38a)]\!] \Leftrightarrow [\![(38b)]\!]$)

The claim in (36), together with the antiexhaustivity implicature entails that Sandy has the right to have this cake and the right to have that ice cream.[7]

The same reasoning derives the distribution requirement in the necessity cases. Consider again the second scenario we went over:

(41)
a. Dad: "We have to clean our bedrooms, paint the barn, and mow the lawn."
b. Dad, to Leonor:"You must clean your bedroom."
c. Dad, to Sandy: "You must either clean your bedroom or mow the lawn."

The sentence in (41c) expresses the proposition in (42).

(42) $$\{\lambda w.\forall w' : w' \in \mathscr{D}_w.\exists p \in \left\{ \begin{array}{c} \lambda w.\mathbf{clean}_w(\mathbf{sandy}, \mathbf{the\text{-}bedroom}), \\ \lambda w.\mathbf{mow}_w(\mathbf{sandy}, \mathbf{the\text{-}lawn}) \end{array} \right\} \;\&\; p(w') = 1\}$$

Why did Dad choose to utter (41c), instead of the stronger (43a) or (43b)?

(43)
a. Sandy must clean her bedroom.
$[\![\text{must}(\exists(\text{Sandy clean her bedroom}))]\!] =$
$\{\lambda w.\forall w' : w' \in \mathscr{D}_w.\exists p \in \{\lambda w.\mathbf{clean}_w(\mathbf{sandy}, \mathbf{the\text{-}bedroom})\} \;\&\; p(w') = 1\}$
b. Sandy must mow the lawn.
$[\![\text{must}(\exists(\text{Sandy mow the lawn}))]\!] =$
$\{\lambda w.\forall w' : w' \in \mathscr{D}_w.\exists p \in \{\lambda w.\mathbf{mow}_w(\mathbf{sandy}, \mathbf{the\text{-}lawn})\} \;\&\; p(w') = 1\}$

Upon hearing (41b), Leonor concluded that she was not required to paint the barn or mow the lawn — she assumes that if Dad would have wanted her to paint the barn or mow the lawn, he would have explicitly said so. Uttering (43a) could have then led Sandy to draw the exhaustivity inference that she was not required to mow the lawn. By widening the domain, Dad wants to avoid the exhaustivity inference that (43b) is false: we conclude that if (43a) is true, then so must be (43b). Parallel reasoning leads to the conclusion that if (43b) is true, then so must be (43a).

(44) (40c) $+>$ ($[\![(42a)]\!] \Leftrightarrow [\![(42b)]\!]$)

The claim in (42) together with the implicature in (44) entails that Sandy is permitted to clean her bedroom and that she is also permitted to mow the lawn. For assume (42) is true. Then if (43a) is false, given (44), (43b) must be false. That contradicts the assumption that (42) is true. The distribution requirement is delivered.

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[6] Notation: the symbol "$+>$" stands for "conversationally implicates" (Levinson, 2001).

[7] A word of caution. One could conclude that, by parity of reasoning, unembedded disjunctions should be interpreted as conjunctions. In this system, in the case of apparent unembedded disjunctions, the alternatives are caught by an implicit epistemic universal. The disjuncts are distributed over the epistemic space: each must be an epistemic possibility.

5. To conclude

Analysis 2 delivers the distribution requirement for both the *may* and *must* cases. None of the objections raised against Analysis 1 apply. First, downward entailing environments are no longer a threat. In negative environments widening the domain does strengthen the assertion and, so, the antiexhaustivity implicature, if at all drawn, goes undetected — as it usually happens with the quantity implicatures drawn in positive environments (Levinson, 2001). Second, since the propositional alternatives are routinely caught by an intervening existential, we can still assume that if, in fact, *either...or* requires Existential Closure, its needs are satisfied in the construction at stake.

To conclude, let me point out that there is an old objection to deriving the distribution requirement as a conversational implicature (Kamp, 1978). The objection relies on the assumption that conversational implicatures are global, that they involve reasoning about a whole utterance, not just parts thereof. If that is the case — the objection goes — examples like the conditional in (45) (modelled after one in Kamp (1978)) pose, apparently, a problem. The conditional seems to range over worlds where Sandy has the right to borrow *Moby Dick* and also the right to borrow *Huckleberry Finn*. The implicature that (46a) is true if and only if (46b) is, together with the claim that Sandy must be pleased if she has at least one of the two rights, does not capture the intended reading (the total meaning could be true if Sandy does not have to be pleased if she has both rights).

(45) Usually, Sandy may have only cake, so if she may have cake or ice cream now, she must be really pleased. (Upon Kamp (1978)).

(46) a. If Sandy may have cake, she must be really pleased.

b. If Sandy may have ice cream, she must be really pleased.

The range of constructions where the distribution requirement *seems* to be local hasn't been studied in any depth yet, and so it still remains to be seen whether Analysis 2 should be implemented in a framework where the implicature can be computed locally. But that goes well beyond the scope of this short paper.

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